Week 1: Assignment 1

The due date for submitting this assignment has passed.

Due on 2021-08-18, 23:59 IST.

As per our records you have not submitted this assignment.

1) Let X be a set and let $d : X \times X \to \mathbb{R}$ (*not* $\mathbb{R}^+ \cup \{0\}$) be a function that satisfies the three conditions in the definition of metric. Then *1 point*

- (X, d) is always a metric space.
- (X, d) need not a metric space but $(X, |d|)$ is a metric space.
- It is possible that neither $(X, |d|)$ nor (X, d) is a metric space.
- Both (X, d) and $(X, |d|)$ are metric spaces.

No, the answer is incorrect.
Score: 0

Accepted Answers:
(X, d) is always a metric space.
Both (X, d) and $(X, |d|)$ are metric spaces.

2) Let X be a metric space. Mark the true statements *1 point*

- There always exists a continuous function $f : X \to Y$ where Y is any other metric space.
- If the metric on X is the discrete metric then **any** function $f : X \to Y$ where Y is any metric space is continuous.
- If metric on Y is the discrete metric then no function $f : X \to Y$ is continuous.
- If $f : X \to Y$ is continuous where Y is metric space then f is still continuous if the metrics on both X and Y are changed to some other equivalent metrics.

No, the answer is incorrect.
Score: 0

Accepted Answers:
There always exists a continuous function $f : X \to Y$ where Y is any other metric space.
*If the metric on X is the discrete metric then **any** function $f : X \to Y$ where Y is any metric space is continuous.*
If $f : X \to Y$ is continuous where Y is metric space then f is still continuous if the metrics on both X and Y are changed to some other equivalent metrics.

3) Continuity of functions between metric spaces can be characterised using which of the following: *1 point*

- Open sets
- Closed sets
- Seqeunces
- Bounded sets

No, the answer is incorrect.
Score: 0

Accepted Answers:
Open sets
Closed sets
Seqeunces

4) Which of the following about closed and open balls in a metric space are true? Here by closed ball, we mean the a set of the form *1 point*

$$\{y : d(x, y) \leq r\}.$$

- Any open set is a union of open balls
- Any closed set is a finite union of closed balls.
- The closed unit ball of radius r is the closure of the open unit ball of radius r.
- The interior of the closed unit ball of radius r is the open unit ball of radius r.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Any open set is a union of open balls

5) Which of the following are true *1 point*

- All norms give rise to metrics.
- All inner-product give rise to norms.
- All norms come from inner-products
- All norms on $\mathbb{R}^n$ give the same metric.

No, the answer is incorrect.
Score: 0

Accepted Answers:
All norms give rise to metrics.
All inner-product give rise to norms.

6) In which of the following metrics on $\mathbb{R}$ does the sequence $\frac{1}{n}$ converge to 0? *1 point*

- Euclidean metric.
- The discrete metric.
- The metric given by

$$d(x, y) := \frac{|x-y|}{|x-y|+1}$$

- The metric given by

$$d(x, y) = c|x - y|,$$

where $c > 0$ is a fixed constant.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Euclidean metric.
The metric given by

$$d(x, y) := \frac{|x-y|}{|x-y|+1}$$

The metric given by

$$d(x, y) = c|x - y|,$$

where $c > 0$ is a fixed constant.

7) Let X be a metric space and consider the function diam : $\mathcal{P}(X) \to \mathbb{R}^+ \cup \{0\}$ defined by *1 point*

diam(A):= the diameter of the set A.

Which of the following subsets of $\mathbb{R}^+ \cup \{0\}$ **cannot** possibly be the range of this function?

- $\{0\}$
- $\{0, 1\}$
- $\mathbb{N}$
- $\mathbb{R}^+ \cup \{0\}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\mathbb{N}$

8) Let $\mathbb{R}^2$ be given some norm. Which of the following sets **cannot** be the open unit ball centered at 0 under this norm? *0 points*

- $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}.$
- $[-1, 1] \times [-1, 1]$
- The unit circle centered at 0.
- $[-1, 1] \times \{0\}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
The unit circle centered at 0.
$[-1, 1] \times \{0\}$