Unit 10 - Week 8

Assignment 8

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-11-11, 23:59 IST.

1) Let f be a differentiable function defined on the interval $[0,3]$, and assume that $f(0)=1$, $f(1)=2$, and $f(3)=2$. Which of the following are true? *1 point*

- We can find $d \in [0,3]$ such that $f(d)=d$.
- It is impossible that $f(d)=d$ for any choice of $d \in [0,3]$.
- We can find $c \in [0,3]$ such that $f'(c)=\frac{1}{3}$.
- We can find $c \in [0,3]$ such that $f'(c)=\frac{1}{4}$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
We can find $d \in [0,3]$ such that $f(d)=d$
We can find $c \in [0,3]$ such that $f'(c)=\frac{1}{3}$.
We can find $c \in [0,3]$ such that $f'(c)=\frac{1}{4}$.

2) Let f be continuous on the open interval $(-1,1)$. Which of the following are true? *1 point*

- If f is differentiable in $(-1,1)$ then $f'(0)=\lim_{c\to 0}\frac{f(c)-f(0)}{c}$
- If f is differentiable in $(-1,1)$ then $f'(0)=\lim_{c\to 0}\frac{f(c)-f(-c)}{2c}$
- If f is differentiable for all $x \in (-1,1)\setminus\{0\}$ and if $\lim_{x\to 0} f'(x)=L$ then f is differentiable at 0 and $f'(0)=L$.
- If f is twice differentiable in $(-1,1)$ and f'' is continuous in $(-1,1)$ then
$$f''(0)=\lim_{h\to 0}\frac{f(h)-2f(0)+f(-h)}{h^2}$$

No, the answer is incorrect.
Score: 0
Accepted Answers:
If f is differentiable in $(-1,1)$ then $f'(0)=\lim_{c\to 0}\frac{f(c)-f(0)}{c}$
If f is differentiable in $(-1,1)$ then $f'(0)=\lim_{c\to 0}\frac{f(c)-f(-c)}{2c}$
If f is differentiable for all $x \in (-1,1)\setminus\{0\}$ and if $\lim_{x\to 0} f'(x)=L$ then f is differentiable at 0 and $f'(0)=L$.
If f is twice differentiable in $(-1,1)$ and f'' is continuous in $(-1,1)$ then
$$f''(0)=\lim_{h\to 0}\frac{f(h)-2f(0)+f(-h)}{h^2}$$

3) We say a function $f:[a,b]\to\mathbb{R}$ is Lipschitz if we can find $C>0$ such that $\forall x, y \in [a,b]$, we have: *1 point*

$$|f(x)-f(y)| \le C|x-y|.$$

Determine which of the following statements are true:

- Any Lipschitz function is uniformly continuous
- Any C^1 -smooth function is Lipschitz.
- Any Lipschitz function is continuous.
- Any Lipschitz function is C^1 -smooth.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Any Lipschitz function is uniformly continuous
Any C^1 -smooth function is Lipschitz.
Any Lipschitz function is continuous.

4) Let $P(x)$ be a degree $n>1$ polynomial that has exactly n distinct real roots. Then: *1 point*

- The derivative $P'(x)$ has exactly $n-1$ distinct real roots.
- All the roots of $P'(x)$ are real but there might not be $n-1$ distinct roots.
- All the roots of $P'(x)$ need not always be real.
- If all roots of $P'(x)$ are real then there are exactly $n-1$ distinct roots.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The derivative $P'(x)$ has exactly $n-1$ distinct real roots.
If all roots of $P'(x)$ are real then there are exactly $n-1$ distinct roots.

5) Which of the following were used in the proof of L'Hôpital's rule? *1 point*

- Darboux's theorem.
- Fundamental theorem of calculus.
- The ratio mean value theorem.
- Cauchy's mean value theorem.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The ratio mean value theorem.
Cauchy's mean value theorem.

6) Let $f, g:\mathbb{R}\to\mathbb{R}$ be functions. Which of the following is possible. *1 point*

- f and g are both not differentiable at 0 but fg is differentiable at 0
- f is not differentiable at 0 but both g and fg are differentiable at 0
- f is differentiable at 0 but at no other point
- f is not differentiable at 0 but g and $f+g$ are differentiable at 0

No, the answer is incorrect.
Score: 0
Accepted Answers:
f and g are both not differentiable at 0 but fg is differentiable at 0
f is not differentiable at 0 but both g and fg are differentiable at 0
f is differentiable at 0 but at no other point

7) Let $f:\mathbb{R}\to\mathbb{R}$ be defined by: *1 point*

$$f(x)=\begin{cases}\frac{x}{2}+x^2\sin\left(\frac{1}{x}\right) & x\neq 0\\ 0 & x=0\end{cases}$$

Then

- f is not differentiable at 0.
- f is continuous everywhere.
- f is differentiable at 0 and $f'(0)>0$.
- f is increasing in some suitably small interval around 0.

No, the answer is incorrect.
Score: 0
Accepted Answers:
f is continuous everywhere.
f is differentiable at 0 and $f'(0)>0$.

8) Which of the following properties about a function $f:[a,b]\to\mathbb{R}$ is guaranteed if it is differentiable. *1 point*

- f is continuous in $[a,b]$
- f is injective.
- f' satisfies the intermediate value property
- f satisfies the intermediate value property

No, the answer is incorrect.
Score: 0
Accepted Answers:
f is continuous in $[a,b]$
f' satisfies the intermediate value property
f satisfies the intermediate value property

9) Which of the following were used in the proof of Rolle's theorem? *1 point*

- The extreme value theorem.
- Darboux's theorem.
- The fact that the derivative at a point of maximum of a differentiable function defined on (a,b) is 0
- The fact that if the derivative is always > 0 then the function is strictly increasing.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The extreme value theorem.
The fact that the derivative at a point of maximum of a differentiable function defined on (a,b) is 0

10) Let $p(x)$ be a non-constant polynomial. Then *1 point*

- If the degree of p is odd then p must have a real root.
- If the degree of p is even and the coefficient of the degree n term is negative then p must have a real root.
- If degree of p is even and the constant coefficent is negative then p has a real root.
- Given any natural number $n>1$, we can always find a polynomial of degree n that has no real roots.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If the degree of p is odd then p must have a real root.