Unit 8 - Week 6

Course outline

How does an NPTEL online course work?

Week 0

Week 1

Week 2

Week 3

Week 4

Week 5

Week 6

- WEEK 6 - INTRODUCTION
- 16.1 Language for limits
- 16.2 Infinite limits
- 16.3 One sided limits
- 16.4 Limits of polynomials
- 17.1 Compactness
- 17.2 The Heine-Borel theorem
- 17.3 Open covers and compactness
- 17.4 Equivalent notions of compactness
- 18.1 The extreme value theorem
- 18.2 Uniform continuity
- Week 6 Lecture materials
- Quiz : Assignment 6
- Week 6 Feedback Form : Real Analysis I

Week 7

Week 8

Week 9

Week 10

Week 11

Week 12

Video Download

Live sessions

Text Transcripts

Assignment 6

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-10-28, 23:59 IST.

1) Mark the true statements *1 point*

- ☐ The function $\sqrt{x} : [0, \infty) \to [0, \infty)$ is uniformly continuous.
- ☐ Any bounded and continuous function $f : (a, b) \to \mathbb{R}$ is uniformly continuous.
- ☐ The product of two uniformly continuous functions is uniformly continuous.
- ☐ The function sin : $\mathbb{R} \to \mathbb{R}$ is uniformly continuous.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The function $\sqrt{x} : [0, \infty) \to [0, \infty)$ is uniformly continuous.
The function sin : $\mathbb{R} \to \mathbb{R}$ is uniformly continuous.

2) Let $f, g : \mathbb{R} \to \mathbb{R}$ be continuous functions. Which of the following are continuous? *1 point*

- ☐ $max\{f, g\}$.
- ☐ $min\{f, g\}$.
- ☐ $\sqrt{f^2}g$.
- ☐ $f \circ g^2$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$max\{f, g\}$.
$min\{f, g\}$.
$\sqrt{f^2}g$.
$f \circ g^2$.

3) Which of the following statements about compact sets are true? *1 point*

- ☐ If K is compact then it contains both its supremum and infimum.
- ☐ If a set K contains both its supremum and infimum then it is compact.
- ☐ Any closed subset of a compact set is compact.
- ☐ K is closed and bounded.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If K is compact then it contains both its supremum and infimum.
Any closed subset of a compact set is compact.
K is closed and bounded.

4) Let x_n be a sequence and consider the set $S := \{x_n : n \in \mathbb{N}\}$. *1 point*

- ☐ S can never be a compact set.
- ☐ S is a compact set if and only if it is finite.
- ☐ If x_n converges then S cannot be compact.
- ☐ If x_n converges then S is compact.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If x_n converges then $\overline{S}$ is compact.

5) Let A and B be compact sets. Then *1 point*

- ☐ $A \cup B$ is compact.
- ☐ $A \cap B$ is compact.
- ☐ $A + B := \{a + b : a \in A, b \in B\}$ is compact.
- ☐ The set $A \backslash B$ can never be compact.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$A \cup B$ is compact.
$A \cap B$ is compact.
$A + B := \{a + b : a \in A, b \in B\}$ is compact.

6) Let $f(x) = |x|$. Then *1 point*

- ☐ f is continuous on the whole of $\mathbb{R}$
- ☐ The function f^2 is continuous on the whole of $\mathbb{R}$.
- ☐ The function f is not continuous at 0
- ☐ The function $\sqrt{f(x)}$ is continuous on the whole of $\mathbb{R}$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
f is continuous on the whole of $\mathbb{R}$
The function f^2 is continuous on the whole of $\mathbb{R}$.
The function $\sqrt{f(x)}$ is continuous on the whole of $\mathbb{R}$.

7) Let K and L be two disjoint non-empty sets in $\mathbb{R}$ and define *1 point*
$d = inf\{|x - y| : x \in K, y \in L\}$.
Which of the following statements are true about d?

- ☐ If K and L are closed sets then $d > 0$.
- ☐ Suppose $d > 0$. Then we can find $k \in K$ and $l \in L$ such that $d = |k - l|$.
- ☐ If K and L are compact then $d > 0$.
- ☐ If K and L are compact then we can find $k \in K$ and $l \in L$ such that $d = |k - l|$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If K and L are compact then $d > 0$.
If K and L are compact then we can find $k \in K$ and $l \in L$ such that $d = |k - l|$.

8) Consider the set (0, 1) which we know is non-compact. Consider the open cover of (0, 1) given by *1 point*

$$O := \left\{ \left(\frac{1}{n}, 1\right) : n \in \mathbb{N} \right\} \cup \{(-1, 0.1)\}$$

Which of the following are true?

- ☐ We cannot find a finite subcover of O that covers (0, 1) as (0, 1) is not compact.
- ☐ The smallest finite subcover of O has 12 elements.
- ☐ The cover O does not have a Lebesgue number with respect to (0, 1).
- ☐ Any number less than 1 is a Lebesgue number for O with respect to (0, 1).

No, the answer is incorrect.
Score: 0
Accepted Answers:
The cover O does not have a Lebesgue number with respect to (0, 1).

9) Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Which of the following English statements are appropriate ways of stating that $\lim_{x \to \infty} f(x) = 0$? *1 point*

- ☐ $|f(x)|$ can be made arbitrarily small when x is sufficiently close to ∞.
- ☐ $|f(x)|$ can be made arbitrarily small when x is arbitrarily large.
- ☐ $|f(\frac{1}{x})|$ can be made arbitrarily small when x is sufficiently close to 0 but not 0.
- ☐ $|f(\frac{1}{x^2})|$ can be made arbitrarily small when x is sufficiently close to 0 but not 0.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$|f(x)|$ can be made arbitrarily small when x is arbitrarily large.
$|f(\frac{1}{x^2})|$ can be made arbitrarily small when x is sufficiently close to 0 but not 0.

10) Let $f : A \to \mathbb{R}$ be continuous. Which of the following statements are true about $B := f(A)$? *1 point*

- ☐ If A is open then B is open.
- ☐ If A is closed then B is closed.
- ☐ If A is closed and bounded then B is also closed and bounded.
- ☐ If A is unbounded then B is unbounded.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If A is closed and bounded then B is also closed and bounded.