

## Unit 7 - Week 5

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<input type="radio"/> 13.4 Basic properties of open and closed sets
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## Assignment 5

The due date for submitting this assignment has passed.  
As per our records you have not submitted this assignment.

**Due on 2020-10-21, 23:59 IST.**

1) Mark the true statements 1 point

- Any union of open sets is open  
 The complement of an open set can never be open  
 Any subset of  $\mathbb{R}$  is either open or closed  
 A subset of  $\mathbb{R}$  can never be both open and closed

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
Any union of open sets is open

2) Which of the following is true about  $\mathbb{Q}$ ? 1 point

- Each point of  $\mathbb{N}$  is an isolated point of  $\mathbb{Q}$   
 The set of limit points of  $\mathbb{Q}$  is  $\mathbb{R}$   
 The set of limit points of  $\mathbb{R} \setminus \mathbb{Q}$  is empty  
 Any adherent point of  $\mathbb{Q}$  is also a limit point

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
The set of limit points of  $\mathbb{Q}$  is  $\mathbb{R}$   
Any adherent point of  $\mathbb{Q}$  is also a limit point

3) Suppose your task is to prove that a set  $U \subset \mathbb{R}$  is open. Which of the following are plausible strategies? 1 point

- Show that every point of  $\mathbb{R} \setminus U$  is an isolated point of  $U$   
 Show that every point of  $\mathbb{R} \setminus U$  is an isolated point of  $\mathbb{R} \setminus U$   
 Show that each point of  $\mathbb{R} \setminus U$  is an interior point of  $\mathbb{R} \setminus U$   
 Show that each point of  $U$  is an interior point of  $U$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
Show that each point of  $U$  is an interior point of  $U$

4) Let  $x_n$  be a sequence and consider the set  $S := \{x_n : n \in \mathbb{N}\}$  1 point

- $S$  can never be an open set  
  $S$  is a closed set if and only if it is finite  
 If  $x_n$  converges then  $S$  cannot be closed  
 If  $x_n$  is a bounded sequence then  $S$  is closed

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $S$  can never be an open set

5) Let  $f, g : \mathbb{R} \rightarrow \mathbb{R}$  be functions. Which one of the following is possible? 1 point

- $f$  and  $f + g$  are continuous at 0 but  $fg$  is not continuous at 0  
  $f$  and  $g$  are not continuous at 0 but  $f + g$  is  
  $f$  is not continuous at 0 but  $g \circ f$  is.  
 Neither  $f$  nor  $g$  are continuous at 0 but  $f + g$  is

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $f$  and  $g$  are not continuous at 0 but  $f + g$  is  
 $f$  is not continuous at 0 but  $g \circ f$  is.  
Neither  $f$  nor  $g$  are continuous at 0 but  $f + g$  is

6) Consider the absolute value function  $f(x) := |x|$  defined on  $\mathbb{R}$ . Then 1 point

- $f$  is continuous on the whole of  $\mathbb{R}$   
 The function  $f^2$  is continuous on the whole of  $\mathbb{R}$ .  
 The function  $f$  is not continuous at 0  
 The function  $\sqrt{f(x)}$  is continuous on the whole of  $\mathbb{R}$ .

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $f$  is continuous on the whole of  $\mathbb{R}$   
The function  $f^2$  is continuous on the whole of  $\mathbb{R}$ .  
The function  $\sqrt{f(x)}$  is continuous on the whole of  $\mathbb{R}$ .

7) Suppose you are given a function  $f : \mathbb{R} \rightarrow \mathbb{R}$  and are asked to show that  $f$  is not continuous at 0. Which of the following is a plausible strategy? 1 point

- Show that the sequence  $f(\frac{1}{n})$  does not converge to  $f(0)$   
 Show that  $f$  cannot be obtained by adding, subtracting, multiplying and composing functions that are known to be continuous  
 Show that  $\lim_{x \rightarrow 0} f(x)$  does not exist.  
 Show that  $\lim_{x \rightarrow 0} f(x) \neq f(0)$ .

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
Show that the sequence  $f(\frac{1}{n})$  does not converge to  $f(0)$   
Show that  $\lim_{x \rightarrow 0} f(x)$  does not exist.  
Show that  $\lim_{x \rightarrow 0} f(x) \neq f(0)$ .

8) Consider the function 1 point

$$f(x) := \begin{cases} g(x) \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Then for which choices of the function  $g : \mathbb{R} \rightarrow \mathbb{R}$  is  $f$  continuous?

- $x$   
  $x^2$   
  $|x|$   
 1

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $x$   
 $x^2$   
 $|x|$

9) Which of the following identities about closure and interior are true? 1 point

- $\overline{A \cup B} = \overline{A} \cup \overline{B}$   
  $\overline{A \cap B} = \overline{A} \cap \overline{B}$   
  $\mathbb{R} \setminus \overline{E} = \text{int}(\mathbb{R} \setminus E)$   
  $\text{int}(E) = \overline{\mathbb{R} \setminus E}$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $\overline{A \cup B} = \overline{A} \cup \overline{B}$   
 $\mathbb{R} \setminus \overline{E} = \text{int}(\mathbb{R} \setminus E)$

10) Which of the following requests are possible? 1 point

- A countable subset of  $[0, 1]$  that has no limit points  
 A countable subset of  $[0, 1]$  which is closed  
 A countable subset of  $[0, 1]$  that has no isolated points  
 An uncountable subset of  $[0, 1]$  that has an isolated point

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
A countable subset of  $[0, 1]$  that has no limit points  
A countable subset of  $[0, 1]$  which is closed  
A countable subset of  $[0, 1]$  that has no isolated points  
An uncountable subset of  $[0, 1]$  that has an isolated point