Unit 7 - Week 5

Assignment 5

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-10-21, 23:59 IST.

1) Mark the true statements *1 point*

- Any union of open sets is open
- The complement of an open set can never be open
- Any subset of $\mathbb{R}$ is either open or closed
- A subset of $\mathbb{R}$ can never be both open and closed

No, the answer is incorrect.
Score: 0
Accepted Answers:
Any union of open sets is open

2) Which of the following is true about $\mathbb{Q}$? *1 point*

- Each point of $\mathbb{N}$ is an isolated point of $\mathbb{Q}$
- The set of limit points of $\mathbb{Q}$ is $\mathbb{R}$
- The set of limit points of $\mathbb{R}\backslash\mathbb{Q}$ is empty
- Any adherent point of $\mathbb{Q}$ is also a limit point

No, the answer is incorrect.
Score: 0
Accepted Answers:
The set of limit points of $\mathbb{Q}$ is $\mathbb{R}$
Any adherent point of $\mathbb{Q}$ is also a limit point

3) Suppose your task is to prove that a set $U \subset \mathbb{R}$ is open. Which of the following are plausible strategies? *1 point*

- Show that every point of $\mathbb{R}\backslash U$ is an isolated point of U
- Show that every point of $\mathbb{R}\backslash U$ is an isolated point of $\mathbb{R}\backslash U$
- Show that each point of $\mathbb{R}\backslash U$ is an interior point of $\mathbb{R}\backslash U$
- Show that each point of U is an interior point of U

No, the answer is incorrect.
Score: 0
Accepted Answers:
Show that each point of U is an interior point of U

4) Let x_n be a sequence and consider the set $S := \{x_n : n \in \mathbb{N}\}$ *1 point*

- S can never be an open set
- S is a closed set if and only if it is finite
- If x_n converges then S cannot be closed
- If x_n is a bounded sequence then S is closed

No, the answer is incorrect.
Score: 0
Accepted Answers:
S can never be an open set

5) Let $f, g : \mathbb{R} \to \mathbb{R}$ be functions. Which one of the following is possible? *1 point*

- f and $f+g$ are continuous at 0 but fg is not continuous at 0
- f and g are not continuous at 0 but $f+g$ is
- f is not continuous at 0 but $g \circ f$ is.
- Neither f nor g are continuous at 0 but $f+g$ is

No, the answer is incorrect.
Score: 0
Accepted Answers:
f and g are not continuous at 0 but $f+g$ is
f is not continuous at 0 but $g \circ f$ is.
Neither f nor g are continuous at 0 but $f+g$ is

6) Consider the absolute value function $f(x) := |x|$ defined on $\mathbb{R}$. Then *1 point*

- f is continuous on the whole of $\mathbb{R}$
- The function f^2 is continuous on the whole of $\mathbb{R}$.
- The function f is not continuous at 0
- The function $\sqrt{f(x)}$ is continuous on the whole of $\mathbb{R}$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
f is continuous on the whole of $\mathbb{R}$
The function f^2 is continuous on the whole of $\mathbb{R}$.
The function $\sqrt{f(x)}$ is continuous on the whole of $\mathbb{R}$.

7) Suppose you are given a function $f : \mathbb{R} \to \mathbb{R}$ and are asked to show that f is not continuous at 0. Which of the following is a plausible strategy? *1 point*

- Show that the sequence $f(\frac{1}{n})$ does not converge to $f(0)$
- Show that f cannot be obtained by adding, subtracting, multiplying and composing functions that are known to be continuous
- Show that $\lim_{x\to 0} f(x)$ does not exist.
- Show that $\lim_{x\to 0} f(x) \neq f(0)$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Show that the sequence $f(\frac{1}{n})$ does not converge to $f(0)$
Show that $\lim_{x\to 0} f(x)$ does not exist.
Show that $\lim_{x\to 0} f(x) \neq f(0)$.

8) Consider the function *1 point*

$$f(x) := \begin{cases} g(x)\sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Then for which choices of the function $g : \mathbb{R} \to \mathbb{R}$ is f continuous?

- x
- x^2
- $|x|$
- 1

No, the answer is incorrect.
Score: 0
Accepted Answers:
x
x^2
$|x|$

9) Which of the following identities about closure and interior are true? *1 point*

- $\overline{A \cup B} = \overline{A} \cup \overline{B}$
- $\overline{A \cap B} = \overline{A} \cap \overline{B}$
- $\mathbb{R}\backslash\overline{E} = \mathrm{int}(\mathbb{R}\backslash E)$
- $\mathrm{int}(E) = \overline{\mathbb{R}\backslash E}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\overline{A \cup B} = \overline{A} \cup \overline{B}$
$\mathbb{R}\backslash\overline{E} = \mathrm{int}(\mathbb{R}\backslash E)$

10) Which of the following requests are possible? *1 point*

- A countable subset of [0, 1] that has no limit points
- A countable subset of [0, 1] which is closed
- A countable subset of [0, 1] that has no isolated points
- An uncountable subset of [0, 1] that has an isolated point

No, the answer is incorrect.
Score: 0
Accepted Answers:
A countable subset of [0, 1] that has no limit points
A countable subset of [0, 1] which is closed
A countable subset of [0, 1] that has no isolated points
An uncountable subset of [0, 1] that has an isolated point