Unit 6 - Week 4

Assignment 4

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-10-14, 23:59 IST.

1) Mark the true statements *1 point*

- [ ] The root test and ratio test are tests for absolute convergence.
- [ ] Neither the root test nor the ratio test can show that $\sum \frac{1}{n^2}$ converges.
- [ ] The root test can show convergence of $\sum \frac{1}{n^2}$ but the ratio test cannot.
- [ ] Neither the root test nor the ratio test can show that $\sum \frac{1}{n}$ diverges.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The root test and ratio test are tests for absolute convergence.
Neither the root test nor the ratio test can show that $\sum \frac{1}{n^2}$ converges.
Neither the root test nor the ratio test can show that $\sum \frac{1}{n}$ diverges.

2) Which of the following are true? *1 point*

- [ ] Every bounded sequence of real numbers has a convergent subsequence.
- [ ] A monotone sequence of real numbers is convergent if and only if it is bounded.
- [ ] A sequence x_n is convergent if and only if the sequence $|x_n|$ is.
- [ ] Every sequence has a monotone subsequence.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Every bounded sequence of real numbers has a convergent subsequence.
A monotone sequence of real numbers is convergent if and only if it is bounded.
Every sequence has a monotone subsequence.

3) Consider the sequence $a_n := 1/n$. Mark all statements below that are true. *1 point*

- [ ] $\sum_{n=1}^{\infty} a_n$ converges
- [ ] $\sum_{n=1}^{\infty} (\sin n)a_n^2$ converges
- [ ] $\sum_{n=1}^{\infty} \frac{a_n}{(-1)^n}$ converges
- [ ] $\sum_{n=1}^{\infty} a_n^2$ converges

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\sum_{n=1}^{\infty} (\sin n)a_n^2$ *converges*
$\sum_{n=1}^{\infty} \frac{a_n}{(-1)^n}$ *converges*
$\sum_{n=1}^{\infty} a_n^2$ *converges*

4) Let x_n be an increasing sequence of irrational numbers in [0, 2]. Then *1 point*

- [ ] x_n converges to 2
- [ ] x_n converges to $\sqrt{2}$
- [ ] x_n converges to some number in [0, 2]
- [ ] x_n might not be convergent.

No, the answer is incorrect.
Score: 0
Accepted Answers:
x_n *converges to some number in [0, 2]*

5) Consider the assertion: *The sequence* x_n has a convergent subsequence. If x_n are all elements of which of the following sets is the assertion always true. *1 point*

- [ ] [0, ∞)
- [ ] [0, 1]
- [ ] (−1, −1/2)
- [ ] [0, 1] ∪ (−1, −1/2).

No, the answer is incorrect.
Score: 0
Accepted Answers:
[0, 1]
(−1, −1/2)
[0, 1] ∪ (−1, −1/2).

6) Which of the following requests is possible? *1 point*

- [ ] A sequence that has a subsequence that is bounded but contains no convergent subsequence.
- [ ] A sequence that does not contain 0 or 1 as terms but has subsequences that converging to each of these values.
- [ ] A sequence that contains subsequences converging to every point in the infinite set {1, 1/2, 1/3, . . . }.
- [ ] A sequence that contains subsequences converging to every point in the infinite set {1, 1/2, 1/3, . . . } but no subsequence converging to 0.

No, the answer is incorrect.
Score: 0
Accepted Answers:
A sequence that does not contain 0 or 1 as terms but has subsequences that converging to each of these values.
A sequence that contains subsequences converging to every point in the infinite set {1, 1/2, 1/3, . . . }.

7) Which of the following propistions are true? *1 point*

- [ ] If every subsequence of a sequence converges then the sequence converges.
- [ ] If a sequence has a divergent subsequence then the sequence itself is divergent.
- [ ] If a sequence is bounded and divergent then there are two subsequences that converge to different limits.
- [ ] If a sequence is monotone and has a convergent subseqeunce then the sequence itself is convergent.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If every subsequence of a sequence converges then the sequence converges.
If a sequence has a divergent subsequence then the sequence itself is divergent.
If a sequence is bounded and divergent then there are two subsequences that converge to different limits.
If a sequence is monotone and has a convergent subseqeunce then the sequence itself is convergent.

8) Let a_n and b_n be Cauchy sequences. Which of the following sequences are also Cauchy? *0 points*

- [ ] $|a_n - b_n|$
- [ ] $(-1)^n a_n$
- [ ] $a_n b_n$
- [ ] $[[a_n]]$, where $[[x]]$ refers to the greatest integer less than or equal to x

No, the answer is incorrect.
Score: 0
Accepted Answers:
$|a_n - b_n|$
$(-1)^n a_n$
$a_n b_n$

9) Which of the following series converges? *1 point*

- [ ] $\sum_{n=1}^{\infty} \frac{1}{2^n + n}$
- [ ] $1 - \frac{3}{4} + \frac{4}{6} - \frac{5}{8} + \frac{6}{10} - \frac{7}{12} + \ldots$
- [ ] $1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} + \frac{1}{8} - \frac{1}{9} + \ldots.$
- [ ] $1 - \frac{1}{2^2} + \frac{1}{3} - \frac{1}{4^2} + \frac{1}{5} - \frac{1}{6^2} + \frac{1}{7} - \frac{1}{8^2} + \ldots.$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\sum_{n=1}^{\infty} \frac{1}{2^n + n}$

10) Suppose you are give a series $\sum a_n$ and are asked to prove that it diverges. Which of the following ideas are good to acheive this. *1 point*

- [ ] Show that $\sum |a_n|$ diverges.
- [ ] Show that $a_n \nrightarrow 0$
- [ ] Find a divergent series $\sum b_n$ with $b_n \geq a_n$ and apply the comparison test.
- [ ] Apply the root or ratio tests.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Show that $a_n \nrightarrow 0$
Apply the root or ratio tests.