

Unit 6 - Week 4

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Assignment 4

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-10-14, 23:59 IST.

1) Mark the true statements 1 point

- The root test and ratio test are tests for absolute convergence.

Neither the root test nor the ratio test can show that $\sum \frac{1}{n^2}$ converges.

The root test can show convergence of $\sum \frac{1}{n^2}$ but the ratio test cannot.

Neither the root test nor the ratio test can show that $\sum \frac{1}{n}$ diverges.

No, the answer is incorrect.
Score: 0

Accepted Answers:
The root test and ratio test are tests for absolute convergence.

Neither the root test nor the ratio test can show that $\sum \frac{1}{n^2}$ converges.

Neither the root test nor the ratio test can show that $\sum \frac{1}{n}$ diverges.

2) Which of the following are true? 1 point

- Every bounded sequence of real numbers has a convergent subsequence.

- A monotone sequence of real numbers is convergent if and only if it is bounded.

A sequence x_n is convergent if and only if the sequence $|x_n|$ is.

- Every sequence has a monotone subsequence.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Every bounded sequence of real numbers has a convergent subsequence.
A monotone sequence of real numbers is convergent if and only if it is bounded.
Every sequence has a monotone subsequence.

3) Consider the sequence $a_n := 1/n$. Mark all statements below that are true. 1 point

$\sum_{n=1}^{\infty} a_n$ converges

$\sum_{n=1}^{\infty} (\sin n)a_n^2$ converges

$\sum_{n=1}^{\infty} \frac{a_n}{(-1)^n}$ converges

$\sum_{n=1}^{\infty} a_n^2$ converges

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $\sum_{n=1}^{\infty} (\sin n)a_n^2$ converges

$\sum_{n=1}^{\infty} \frac{a_n}{(-1)^n}$ converges

$\sum_{n=1}^{\infty} a_n^2$ converges

4) Let x_n be an increasing sequence of irrational numbers in $[0, 2]$. Then 1 point

x_n converges to 2

x_n converges to $\sqrt{2}$

x_n converges to some number in $[0, 2]$

x_n might not be convergent.

No, the answer is incorrect.
Score: 0

Accepted Answers:
 x_n converges to some number in $[0, 2]$

5) Consider the assertion: The sequence x_n has a convergent subsequence. If x_n are all elements of which of the following sets is the assertion always true. 1 point

$[0, \infty)$

$[0, 1]$

$(-1, -1/2)$

$[0, 1] \cup (-1, -1/2)$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $[0, 1]$

$(-1, -1/2)$

$[0, 1] \cup (-1, -1/2)$.

6) Which of the following requests is possible? 1 point

- A sequence that has a subsequence that is bounded but contains no convergent subsequence.

- A sequence that does not contain 0 or 1 as terms but has subsequences that converging to each of these values.

- A sequence that contains subsequences converging to every point in the infinite set $\{1, 1/2, 1/3, \dots\}$.

- A sequence that contains subsequences converging to every point in the infinite set $\{1, 1/2, 1/3, \dots\}$ but no subsequence converging to 0.

No, the answer is incorrect.
Score: 0

Accepted Answers:
A sequence that does not contain 0 or 1 as terms but has subsequences that converging to each of these values.

A sequence that contains subsequences converging to every point in the infinite set $\{1, 1/2, 1/3, \dots\}$.

7) Which of the following propitions are true? 1 point

- If every subsequence of a sequence converges then the sequence converges.

- If a sequence has a divergent subsequence then the sequence itself is divergent.

- If a sequence is bounded and divergent then there are two subsequences that converge to different limits.

- If a sequence is monotone and has a convergent subsequence then the sequence itself is convergent.

No, the answer is incorrect.
Score: 0

Accepted Answers:
If every subsequence of a sequence converges then the sequence converges.

If a sequence has a divergent subsequence then the sequence itself is divergent.

If a sequence is bounded and divergent then there are two subsequences that converge to different limits.

If a sequence is monotone and has a convergent subsequence then the sequence itself is convergent.

8) Let a_n and b_n be Cauchy sequences. Which of the following sequences are also Cauchy? 0 points

$|a_n - b_n|$

$(-1)^n a_n$

$a_n b_n$

$\lfloor a_n \rfloor$, where $\lfloor x \rfloor$ refers to the greatest integer less than or equal to x

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $|a_n - b_n|$

$(-1)^n a_n$

$a_n b_n$

9) Which of the following series converges? 1 point

$\sum_{n=1}^{\infty} \frac{1}{2^n + n}$

$1 - \frac{3}{4} + \frac{4}{6} - \frac{5}{8} + \frac{6}{10} - \frac{7}{12} + \dots$

$1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} + \frac{1}{8} - \frac{1}{9} + \dots$

$1 - \frac{1}{2^2} + \frac{1}{3} - \frac{1}{4^2} + \frac{1}{5} - \frac{1}{6^2} + \frac{1}{7} - \frac{1}{8^2} + \dots$

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $\sum_{n=1}^{\infty} \frac{1}{2^n + n}$

$\sum_{n=1}^{\infty} \frac{1}{2^n + n}$

10) Suppose you are give a series $\sum a_n$ and are asked to prove that it diverges. Which of the following ideas are good to acheive this. 1 point

Show that $\sum |a_n|$ diverges.

Show that $a_n \rightarrow 0$

Find a divergent series $\sum b_n$ with $b_n \geq a_n$ and apply the comparison test.

- Apply the root or ratio tests.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Show that $a_n \rightarrow 0$

Apply the root or ratio tests.