

## Unit 5 - Week 3

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# Assignment 3

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment.

**Due on 2020-10-07, 23:59 IST.**

1) Which of the following sequences converge to 0 (you can assume all the basic properties of logarithms and exponentials that you have learnt in school). **1 point**

- $\frac{1}{\log n}$
- $\frac{1}{n \log n}$
- $\frac{n}{\log n}$
- $\frac{n}{e^n \log n}$

No, the answer is incorrect.  
Score: 0

Accepted Answers:

- $\frac{1}{\log n}$
- $\frac{1}{n \log n}$
- $\frac{n}{e^n \log n}$

2) Which of the following statements is true about the sequences  $a_n$  and  $b_n$  ? **1 point**

- If  $a_n$  and  $b_n$  are both divergent then so is  $a_n + b_n$
- If  $a_n$  and  $b_n$  are both divergent then  $a_n b_n$  is divergent
- If  $a_n$  and  $a_n + b_n$  are both convergent then  $b_n$  is convergent
- If  $a_n$  and  $a_n b_n$  are both convergent then  $b_n$  is convergent

No, the answer is incorrect.  
Score: 0

Accepted Answers:

If  $a_n$  and  $a_n + b_n$  are both convergent then  $b_n$  is convergent

3) We say that a sequence  $(a_n)$  does not converge to  $l$  if **0 points**

- $\forall \epsilon > 0, \forall n_0 \in \mathbb{N}, \forall n \geq n_0$ , we have  $|a_n - l| > \epsilon$ .
- $\forall \epsilon > 0, \forall n_0 \in \mathbb{N}, \exists n \geq n_0$ , such that  $|a_n - l| > \epsilon$ .
- $\exists \epsilon > 0, \forall n_0 \in \mathbb{N}, \exists n \geq n_0$ , such that  $|a_n - l| > \epsilon$ .
- $\exists \epsilon > 0, \forall n_0 \in \mathbb{N}, \forall n \geq n_0$ , we have  $|a_n - l| > \epsilon$ .

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $\exists \epsilon > 0, \forall n_0 \in \mathbb{N}, \exists n \geq n_0$ , such that  $|a_n - l| > \epsilon$ .

4) Suppose  $a_n$  is a sequence such that every rational number is enumerated by  $a_n$  (this means that every rational number appears when we list the elements of  $a_n$ ). Mark the true statements: **1 point**

- It is impossible for  $a_n$  to be convergent.
- It is possible for  $a_n$  to be convergent.
- Some subsequence of  $a_n$  definitely converges.
- It is possible that some subsequence of  $a_n$  converges.

No, the answer is incorrect.  
Score: 0

Accepted Answers:

It is impossible for  $a_n$  to be convergent.  
Some subsequence of  $a_n$  definitely converges.  
It is possible that some subsequence of  $a_n$  converges.

5) Suppose  $a_n$  and  $b_n$  are sequences such that  $a_n = b_n$  for infinitely many choices of  $n \in \mathbb{N}$ . Then: **1 point**

- If  $a_n$  converges then  $b_n$  converges.
- If  $a_n$  diverges then  $b_n$  diverges.
- $a_n$  converges iff  $b_n$  converges.
- If  $a_n$  and  $b_n$  are both convergent then they both converge to the same limit.

No, the answer is incorrect.  
Score: 0

Accepted Answers:

If  $a_n$  and  $b_n$  are both convergent then they both converge to the same limit.

6) This is to test your understanding of the behaviour of the "tail of the sequence". Let  $a_n$  be a sequence. For each fixed  $k \in \mathbb{N}$  define the new sequence  $\{b_n^{(k)} := a_{k+n}\}_{n=1}^{\infty}$  Which one of the following statements are true: **1 point**

- If  $b_n$  is another sequence such that  $a_n = b_n$  for all but finitely many choices of  $n$  then  $a_n$  is convergent iff  $b_n$  is convergent.
- If  $b_n^{(k)}$  is convergent for some choice of  $k$  then  $a_n$  is also convergent.
- If  $a_n$  is convergent then so is each  $b_n^{(k)}$ .
- If  $a_n$  is divergent the so is each  $b_n^{(k)}$ .

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
If  $b_n$  is another sequence such that  $a_n = b_n$  for all but finitely many choices of  $n$  then  $a_n$  is convergent iff  $b_n$  is convergent.

If  $b_n^{(k)}$  is convergent for some choice of  $k$  then  $a_n$  is also convergent.

If  $a_n$  is convergent then so is each  $b_n^{(k)}$ .

If  $a_n$  is divergent the so is each  $b_n^{(k)}$ .

7) Which of the following requests are possible? **1 point**

- A sequence that has no convergent subsequence.
- A sequence that is divergent but such that every subsequence is convergent.
- A sequence such that for each real number  $r$ , there is some subsequence that converges to  $r$
- A sequence that has both a convergent and divergent subsequence.

No, the answer is incorrect.  
Score: 0

Accepted Answers:

A sequence that has no convergent subsequence.  
A sequence such that for each real number  $r$ , there is some subsequence that converges to  $r$   
A sequence that has both a convergent and divergent subsequence.

8) Which of the following sequences are convergent? **1 point**

- $\frac{1}{n^2}$
- $\frac{1}{\sqrt{n}}$
- $(-1)^n + \frac{1}{n}$
- $(-1)^n + \left(\frac{-1}{n} + 1\right)^n$

No, the answer is incorrect.  
Score: 0

Accepted Answers:

- $\frac{1}{n^2}$
- $\frac{1}{\sqrt{n}}$
- $(-1)^n + \left(\frac{-1}{n} + 1\right)^n$

9) Let  $a_n$  be a sequence that takes only integer values. Mark the true statement: **1 point**

- If  $a_n$  converges then the set  $\{a_n : n \in \mathbb{N}\}$  is finite.
- If the set  $\{a_n : n \in \mathbb{N}\}$  is finite then  $a_n$  converges.
- If the set  $\{a_n : n \in \mathbb{N}\}$  is infinite then  $a_n$  diverges.
- If  $a_n$  diverges then the set  $\{a_n : n \in \mathbb{N}\}$  is infinite.

No, the answer is incorrect.  
Score: 0

Accepted Answers:

If  $a_n$  converges then the set  $\{a_n : n \in \mathbb{N}\}$  is finite.  
If the set  $\{a_n : n \in \mathbb{N}\}$  is infinite then  $a_n$  diverges.

10) What is the limit of the sequence  $\frac{e^n}{n^2}$  **1 point**

- 0
- 1
- $e$
- The sequence is divergent.

No, the answer is incorrect.  
Score: 0

Accepted Answers:

The sequence is divergent.