Unit 5 - Week 3

Assignment 3

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-10-07, 23:59 IST.

1) Which of the following sequences converge to 0 (you can assume all the basic properties of logarithms and exponentials that you have learnt in school). *1 point*

- $\frac{1}{log\, n}$
- $\frac{1}{nlog\, n}$
- $\frac{n}{log\, n}$
- $\frac{n}{e^n\, log\, n}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{1}{log\, n}$
$\frac{1}{nlog\, n}$
$\frac{n}{e^n\, log\, n}$

2) Which of the following statements is true about the sequences a_n and b_n ? *1 point*

- If a_n and b_n are both divergent then so is $a_n + b_n$
- If a_n and b_n are both divergent then $a_n b_n$ is divergent
- If a_n and $a_n + b_n$ are both convergent then b_n is convergent
- If a_n and $a_n b_n$ are both convergent then b_n is convergent

No, the answer is incorrect.
Score: 0
Accepted Answers:
If a_n and $a_n + b_n$ are both convergent then b_n is convergent

3) We say that a sequence (a_n) does not converge to l if *0 points*

- $\forall \epsilon > 0, \forall n_0 \in \mathbb{N}, \forall n \geq n_0$, we have $|a_n - l| > \epsilon.$
- $\forall \epsilon > 0, \forall n_0 \in \mathbb{N}, \exists n \geq n_0,$ such that $|a_n - l| > \epsilon.$
- $\exists \epsilon > 0, \forall n_0 \in \mathbb{N}, \exists n \geq n_0$, such that $|a_n - l| > \epsilon$.
- $\exists \epsilon > 0, \forall n_0 \in \mathbb{N}, \forall n \geq n_0,$ we have $|a_n - l| > \epsilon$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\exists \epsilon > 0, \forall n_0 \in \mathbb{N}, \exists n \geq n_0$, *such that* $|a_n - l| > \epsilon$.

4) Suppose a_n is a sequence such that every rational number is enumerated by a_n (this means that every rational number appears when we list the elements of a_n). Mark the true statements: *1 point*

- It is impossible for a_n to be convergent.
- It is possible for a_n to be convergent.
- Some subsequence of a_n definitely converges.
- It is possible that some subsequence of a_n converges.

No, the answer is incorrect.
Score: 0
Accepted Answers:
It is impossible for a_n to be convergent.
Some subsequence of a_n definitely converges.
It is possible that some subsequence of a_n converges.

5) Suppose a_n and b_n are sequences such that $a_n = b_n$ for infinitely many choices of $n \in \mathbb{N}$. Then: *1 point*

- If a_n converges then b_n converges.
- If a_n diverges then b_n diverges.
- a_n converges iff b_n converges.
- If a_n and b_n are both convergent then they both converge to the same limit.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If a_n and b_n are both convergent then they both converge to the same limit.

6) This is to test your understanding of the behaviour of the "tail of the sequence". Let a_n be a sequence. For each fixed $k \in \mathbb{N}$ define the new sequence $\{b_n^{(k)} := a_{k+n}\}_{n=1}^{\infty}$ Which one of the following statements are true: *1 point*

- If b_n is another sequence such that $a_n = b_n$ for all but finitely many choices of n then a_n is convergent iff b_n is convergent.
- If $b_n^{(k)}$ is convergent for some choice of k then a_n is also convergent.
- If a_n is convergent then so is each $b_n^{(k)}$.
- If a_n is divergent the so is each $b_n^{(k)}$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If b_n is another sequence such that $a_n = b_n$ for all but finitely many choices of n then a_n is convergent iff b_n is convergent.
If $b_n^{(k)}$ is convergent for some choice of k then a_n is also convergent.
If a_n is convergent then so is each $b_n^{(k)}$.
If a_n is divergent the so is each $b_n^{(k)}$.

7) Which of the following requests are possible? *1 point*

- A sequence that has no convergent subsequence.
- A sequence that is divergent but such that every subsequence is convegent.
- A sequence such that for each real number r , there is some subsequence that converges to r
- A sequence that has both a convergent and divergent subsequence.

No, the answer is incorrect.
Score: 0
Accepted Answers:
A sequence that has no convergent subsequence.
A sequence such that for each real number r , there is some subsequence that converges to r
A sequence that has both a convergent and divergent subsequence.

8) Which of the following sequences are convergent? *1 point*

- $\frac{1}{n^2}$
- $\frac{1}{\sqrt{n}}$
- $(-1)^n + \frac{1}{n}$
- $(-1)^n + (\frac{-1}{n} + 1)^n$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{1}{n^2}$
$\frac{1}{\sqrt{n}}$
$(-1)^n + (\frac{-1}{n} + 1)^n$

9) Let a_n be a sequence that takes only integer values. Mark the true statement: *1 point*

- If a_n converges then the set $\{a_n : n \in \mathbb{N}\}$ is finite.
- If the set $\{a_n : n \in \mathbb{N}\}$ is finite then a_n converges.
- If the set $\{a_n : n \in \mathbb{N}\}$ is infinite then a_n diverges.
- If a_n diverges then the set $\{a_n : n \in \mathbb{N}\}$ is infinite.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If a_n converges then the set $\{a_n : n \in \mathbb{N}\}$ is finite.
If the set $\{a_n : n \in \mathbb{N}\}$ is infinite then a_n diverges.

10) What is the limit of the sequence $\frac{e^n}{n^2}$ *1 point*

- 0
- 1
- e
- The sequence is divergent.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The sequence is divergent.