Unit 4 - Week 2

Assignment 2

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-09-30, 23:59 IST.

1) Mark the true statements: *1 point*

- ☐ The Archimedean property holds true for $\mathbb{Q}$.
- ☐ The nested intervals property holds true for $\mathbb{Q}$.
- ☐ The set $\mathbb{R} \setminus \mathbb{Q}$ is a field under usual addition and multiplication.
- ☐ The set $\mathbb{R} \setminus \mathbb{Q}$ is uncountable.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The Archimedean property holds true for $\mathbb{Q}$.
The set $\mathbb{R} \setminus \mathbb{Q}$ is uncountable.

2) Let F be a field and $a, b \in F$ such that $a \neq 0$. Then which of the following are true *1 point*

- ☐ The equation $ax = b$ has no solution for x in F.
- ☐ The equation $ax = b$ has at least one solution for x in F.
- ☐ The equation $ax = b$ has exactly one solution for x in F.
- ☐ If F is an infinite field, then $ax = b$ has infinitely many solution for x in F.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The equation $ax = b$ has at least one solution for x in F.
The equation $ax = b$ has exactly one solution for x in F.

3) Which of the following statements are true. *1 point*

- ☐ $\sqrt{2}$ is the supremum of a set which contains only rational numbers.
- ☐ $\sqrt{2}$ is the infimum of a set which contains only rational numbers.
- ☐ $\sqrt{2}$ is the supremum of a set which contains only irrational numbers.
- ☐ $\sqrt{2}$ is the infimum of a set which contains only irrational numbers.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\sqrt{2}$ is the supremum of a set which contains only rational numbers.
$\sqrt{2}$ is the infimum of a set which contains only rational numbers.
$\sqrt{2}$ is the supremum of a set which contains only irrational numbers.
$\sqrt{2}$ is the infimum of a set which contains only irrational numbers.

4) Which of the following subsets of $\mathbb{R}$ are possible: *1 point*

- ☐ A set B with inf $B \geq$ sup B
- ☐ A finite set that contains its infimum but not its supremum.
- ☐ A bounded subset of $\mathbb{Q}$ that contains its supremum but not its infimum.
- ☐ A bounded subset of $\mathbb{N}$ that does not contain its infimum.

No, the answer is incorrect.
Score: 0
Accepted Answers:
A set B with inf $B \geq$ sup B
A bounded subset of $\mathbb{Q}$ that contains its supremum but not its infimum.

5) Let $A, B \subset \mathbb{R}$ and $C = \{a + b : a \in A, b \in B\}$. Which of the following are true? *1 point*

- ☐ If A and B are bounded sets then C is a bounded set.
- ☐ If C is a bounded set, then A and B are bounded sets.
- ☐ If $\mathbb{R} \setminus A$ and $\mathbb{R} \setminus B$ are bounded sets then $\mathbb{R} \setminus C$ is a bounded set.
- ☐ If $\mathbb{R} \setminus C$ is a bounded set, then $\mathbb{R} \setminus A$ and $\mathbb{R} \setminus B$ are bounded sets.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If A and B are bounded sets then C is a bounded set.
If C is a bounded set, then A and B are bounded sets.
If $\mathbb{R} \setminus A$ and $\mathbb{R} \setminus B$ are bounded sets then $\mathbb{R} \setminus C$ is a bounded set.

6) Which of the following requests are possible: *1 point*

- ☐ Two bounded sets A and B with $A \cap B = \emptyset$ and s = sup A = sup B but $s \notin A$ and $s \notin B$.
- ☐ Nested closed intervals $A_1 \supset A_2 \supset A_3 \ldots$ whose intersection contains more than one point.
- ☐ Nested open intervals $A_1 \supset A_2 \supset A_3 \ldots$ whose intersection is a single point.
- ☐ Nested open intervals $A_1 \supset A_2 \supset A_3 \ldots$ whose intersection is the empty set.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Two bounded sets A and B with $A \cap B = \emptyset$ and s = sup A = sup B but $s \notin A$ and $s \notin B$.
Nested closed intervals $A_1 \supset A_2 \supset A_3 \ldots$ whose intersection contains more than one point.
Nested open intervals $A_1 \supset A_2 \supset A_3 \ldots$ whose intersection is a single point.
Nested open intervals $A_1 \supset A_2 \supset A_3 \ldots$ whose intersection is the empty set.

7) Which one of the following statements about the real numbers are true? *1 point*

- ☐ If $x > y$ and $z \neq 0$ then $xz > yz$.
- ☐ If $xy = xz$ then $y = z$.
- ☐ If $0 < x < y$ then $1/x > 1/y$.
- ☐ If $x + y = x + z$ then $y = z$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $0 < x < y$ then $1/x > 1/y$.
If $x + y = x + z$ then $y = z$.

8) Which of the following numbers are irrational. *1 point*

- ☐ $\sqrt{50}$
- ☐ $\sqrt{2} + \sqrt{3}$
- ☐ $2^{\frac{3}{2}}$
- ☐ The sum of any two irrational numbers.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\sqrt{50}$
$\sqrt{2} + \sqrt{3}$
$2^{\frac{3}{2}}$

9) All of us know the famous formula to find the roots of the quadratic equation *1 point*

$$ax^2 + bx + c = 0$$

is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Which of the following methods are plausible ways to prove this formula.

- ☐ Directly substitute the formula in the equation and see that the equation is satisfied.
- ☐ Completing the square.
- ☐ Assume that $y = x^2$ and substitute in the equation to get a linear equation.
- ☐ Find the general formula to solve a polynomial of degree n and then set $n = 2$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Directly substitute the formula in the equation and see that the equation is satisfied.
Completing the square.

10) A real number is said to be algebraic if it is the root of a polynomial all of whose coefficients are integers. Which of the following statements combined together appropriately prove that the set of algebraic numbers is countable. *1 point*

- ☐ Any algebraic number has to be rational.
- ☐ The set of polynomials with integer coefficients is countable.
- ☐ Any non-constant polynomial has only a finite number of roots.
- ☐ Countable union of countable sets is countable.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The set of polynomials with integer coefficients is countable.
Any non-constant polynomial has only a finite number of roots.
Countable union of countable sets is countable.