

## Unit 12 - Week 10

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# Assignment 10

The due date for submitting this assignment has passed.  
As per our records you have not submitted this assignment.

Due on 2020-11-25, 23:59 IST.

1) Let  $f : [0, 1] \rightarrow \mathbb{R}$  be differentiable. Which of the following are true? 1 point

- The fundamental theorem of calculus guarantees that  $f'$  is integrable.
- If  $f'$  is continuous then  $f'$  is integrable.
- If  $f'$  is monotone then  $f'$  is integrable.
- $f'$  is integrable iff  $f'$  is continuous.

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
If  $f'$  is continuous then  $f'$  is integrable.  
If  $f'$  is monotone then  $f'$  is integrable.

2) Let  $f, g : [0, 1] \rightarrow \mathbb{R}$  be integrable. Then 1 point

- $f + g$  is integrable.
- $fg$  is integrable.
- $\frac{1}{f}$  is integrable if  $f$  is never zero.
- $\frac{g}{f}$  is integrable if  $f$  is never zero.

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $f + g$  is integrable.  
 $fg$  is integrable.

3) Let  $f_n : [0, 1] \rightarrow \mathbb{R}$  converge uniformly to  $f : [0, 1] \rightarrow \mathbb{R}$ . Then 1 point

- $f$  is continuous.
- If each  $f_n$  is continuous then  $f$  is continuous.
- If  $f_n$  are differentiable then so is  $f$ .
- If  $f_n$  are integrable then so is  $f$ .

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
If each  $f_n$  is continuous then  $f$  is continuous.  
If  $f_n$  are integrable then so is  $f$ .

4) Which of the following identities involving power series are valid in the set (0, 1). 1 point

- $\frac{1}{1-x} = 1 + x + x^2 + \dots$
- $\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \dots$
- $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 + \dots$
- $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $\frac{1}{1-x} = 1 + x + x^2 + \dots$   
 $\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \dots$   
 $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 + \dots$   
 $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

5) Which of the following sequence of functions  $f_n$  converge uniformly on  $[0, 1]$ ? 1 point

- $f_n = \frac{x}{n}$
- $f_n = \frac{x^2}{n}$
- $f_n = x^n$
- $f_n = \frac{x^n}{n}$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $f_n = \frac{x}{n}$   
 $f_n = \frac{x^2}{n}$   
 $f_n = \frac{x^n}{n}$

6) Which of the following power series have a positive radius of convergence? 1 point

- $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$
- $\sum_{n=0}^{\infty} n!x^n$
- $\sum_{n=0}^{\infty} x^{n!}$
- $\sum_{n=0}^{\infty} n^n x^n$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$   
 $\sum_{n=0}^{\infty} x^{n!}$

7) Which of the following statements about uniformly convergent sequences  $f_n, g_n : \mathbb{R} \rightarrow \mathbb{R}$  of functions are true? 1 point

- The sum  $f_n + g_n$  is uniformly convergent.
- The product  $f_n g_n$  is uniformly convergent.
- If  $|f_n| < M$  and  $|g_n| < M$  for some  $M \in \mathbb{R}$  and for all choices of  $n \in \mathbb{N}$  then  $f_n g_n$  is uniformly convergent.
- $f_n^2$  is uniformly convergent.

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
The sum  $f_n + g_n$  is uniformly convergent.  
If  $|f_n| < M$  and  $|g_n| < M$  for some  $M \in \mathbb{R}$  and for all choices of  $n \in \mathbb{N}$  then  $f_n g_n$  is uniformly convergent.

8) Which of the following were used to prove the fundamental theorem of calculus. 1 point

- The Riemann–Lebesgue theorem.
- The additivity of the integral, i.e.,  $\int_a^b f = \int_a^c f + \int_c^b f$ .
- Lagrange's mean value theorem.
- Darboux's theorem.

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
The additivity of the integral, i.e.,  $\int_a^b f = \int_a^c f + \int_c^b f$ .  
Lagrange's mean value theorem.