Unit 12 - Week 10

Assignment 10

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-11-25, 23:59 IST.

1) Let $f : [0,1] \to \mathbb{R}$ be differentiable. Which of the following are true? *1 point*

- The fundamental theorem of calculus guarantees that f' is integrable.
- If f' is continuous then f' is integrable.
- If f' is monotone then f' is integrable.
- f' is integrable iff f' is continuous.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If f' is continuous then f' is integrable.
If f' is monotone then f' is integrable.

2) Let $f, g : [0,1] \to \mathbb{R}$ be integrable. Then *1 point*

- $f+g$ is integrable.
- fg is integrable.
- $\frac{1}{f}$ is integrable if f is never zero.
- $\frac{g}{f}$ is integrable if f is never zero.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$f+g$ is integrable.
fg is integrable.

3) Let $f_n : [0,1] \to \mathbb{R}$ converge uniformly to $f : [0,1] \to \mathbb{R}$. Then *1 point*

- f is continuous.
- If each f_n is continuous then f is continuous.
- If f_n are differentiable then so is f .
- If f_n are integrable then so is f .

No, the answer is incorrect.
Score: 0
Accepted Answers:
If each f_n is continuous then f is continuous.
If f_n are integrable then so is f .

4) Which of the following identities involving power series are valid in the set (0, 1). *1 point*

- $\frac{1}{1-x} = 1 + x + x^2 + ...$
- $\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + ...$
- $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 + ...$
- $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{1}{1-x} = 1 + x + x^2 + ...$
$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + ...$
$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 + ...$
$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$

5) Which of the following sequence of functions f n converge uniformly on [0, 1]? *1 point*

- $f_n = \frac{x}{n}$
- $f_n = \frac{x^2}{n}$
- $f_n = x^n$
- $f_n = \frac{x^n}{n}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$f_n = \frac{x}{n}$
$f_n = \frac{x^2}{n}$
$f_n = \frac{x^n}{n}$

6) Which of the following power series have a positive radius of convergence? *1 point*

- $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...$
- $\sum_{n=0}^{\infty} n! x^n$
- $\sum_{n=0}^{\infty} x^{n!}$
- $\sum_{n=0}^{\infty} n^n x^n$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...$
$\sum_{n=0}^{\infty} x^{n!}$

7) Which of the following statements about uniformly convergent sequences $f_n, g_n : \mathbb{R} \to \mathbb{R}$ of functions are true? *1 point*

- The sum $f_n + g_n$ is uniformly convergent.
- The product $f_n g_n$ is uniformly convergent.
- If $|f_n| < M$ and $|g_n| < M$ for some $M \in \mathbb{R}$ and for all choices of $n \in \mathbb{N}$ then $f_n g_n$ is uniformly convergent.
- f_n^2 is uniformly convergent.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The sum $f_n + g_n$ is uniformly convergent.
If $|f_n| < M$ and $|g_n| < M$ for some $M \in \mathbb{R}$ and for all choices of $n \in \mathbb{N}$ then $f_n g_n$ is uniformly convergent.

8) Which of the following were used to prove the fundamental theorem of calculus. *1 point*

- The Riemann–Lebesgue theorem.
- The additivity of the integral, i.e., $\int_a^b f = \int_a^c f + \int_c^b f$.
- Lagrange's mean value theorem.
- Darboux's theorem.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The additivity of the integral, i.e., $\int_a^b f = \int_a^c f + \int_c^b f$.
Lagrange's mean value theorem.