Unit 10 - Week 8

Assignment 8

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-11-11, 23:59 IST.

1) Let $f_n : \mathbb{R} \to \mathbb{R}$ be defined as $f_n(x) = sin(n)\chi_{[0,\frac{1}{n}]}(x), n \geq 1$. Then which of the following are true? *1 point*

- $(f_n) \to 0$ in measure
- $(f_n) \to 0$ almost uniformly
- $(f_n) \to 0$ in L^1 norm
- $(f_n) \to 0$ in L^∞ norm

No, the answer is incorrect.
Score: 0
Accepted Answers:
$(f_n) \to 0$ in measure
$(f_n) \to 0$ almost uniformly
$(f_n) \to 0$ in L^1 norm

2) Let $f_n : \mathbb{R} \to \mathbb{R}$ be defined as $f_n = n\chi_{[1-\frac{1}{n},1+\frac{1}{n}]}, n \geq 1$. Then which of the following are true? *1 point*

- $(f_n) \to 0$ pointwise everywhere.
- $(f_n) \to 0$ pointwise almost everywhere.
- $(f_n) \to 0$ uniformly a.e.
- $(f_n) \to 0$ uniformly everywhere.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$(f_n) \to 0$ pointwise almost everywhere.

3) Let $f_n : \mathbb{R} \to \mathbb{R}$ be defined as $f_n(x) = \frac{1}{n}\chi_{[0,\infty)(x)} - \frac{1}{n}\chi_{(-\infty,0]}(x) + \chi_{\{0\}}(x), n \geq 1$. Then which of the following are true? *1 point*

- $(f_n) \to 0$ pointwise everywhere.
- $(f_n) \to 0$ pointwise almost everywhere.
- $(f_n) \to 0$ uniformly a.e.
- $(f_n) \to 0$ uniformly everywhere.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$(f_n) \to 0$ pointwise almost everywhere.
$(f_n) \to 0$ uniformly a.e.

4) Let $f_n : \mathbb{R} \to \mathbb{R}$ be defined as $f_n(x) := \dfrac{sin(\frac{1}{n})}{x^2+n^2}$ Let $f = \sum_{n\geq 1} f_n$. Then *1 point*

- $\sum_{n\geq 1} f_n$ converges absolutely and uniformly everywhere
- The partial sums $g_N := \sum_{n=1}^{N} f_n$ are L^1 functions for each $N \geq 1$
- $f \in L^1(\mathbb{R}, m)$
- $||f - g_N||_{L^1} \to 0$ as $N \to \infty$, where g_N is defined as above.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\sum_{n\geq 1} f_n$ converges absolutely and uniformly everywhere
The partial sums $g_N := \sum_{n=1}^{N} f_n$ are L^1 functions for each $N \geq 1$
$f \in L^1(\mathbb{R}, m)$
$||f - g_N||_{L^1} \to 0$ as $N \to \infty$, where g_N is defined as above.

5) Let $f_n : [0,1] \to \mathbb{R}$ be defined recursively as $f_0(x) = x \forall x \in [0,1]$ *1 point*

$$f_n(x) = \begin{cases} \frac{1}{2} f_{n-1}(3x) & if\ x \in [0,1/3] \\ \frac{1}{2} & if\ x \in (1/3,2/3) \\ \frac{1}{2} + \frac{1}{2} f_{n-1}(3x-2) & if\ x \in [2/3,1] \end{cases}$$

[Hint: Draw the graphs of f_0, f_1, and f_2. Establish an inequality of the form $|f_{n+1} - f_n| \leq \frac{1}{2}|f_n - f_{n-1}|$, using induction.]

- (f_n) converges pointwise everywhere to a function $f : [0,1] \to \mathbb{R}$
- (f_n) converges uniformly everywhere to a function $f : [0,1] \to \mathbb{R}$
- (f_n) converges in L^1 norm to a function $f : [0,1] \to \mathbb{R}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(f_n) converges pointwise everywhere to a function $f : [0,1] \to \mathbb{R}$
(f_n) converges uniformly everywhere to a function $f : [0,1] \to \mathbb{R}$
(f_n) converges in L^1 norm to a function $f : [0,1] \to \mathbb{R}$

6) For a sequence of measurable functions $\{f_n\}, f_n : \mathbb{R} \to \mathbb{R}$ for $n \geq 1$, which of the following are true? *1 point*

- If $f_n \to f$ in L^1 norm, then $f_n \to f$ in measure.
- If $f_n \to f$ in measure, then $f_n \to f$ in L^1 norm
- If $f_n \to f$ in measure, then $f_n \to f$ pointwise a.e.
- If $f_n \to f$ in measure, then $f_n \to f$ almost uniformly.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $f_n \to f$ in L^1 norm, then $f_n \to f$ in measure.

7) For a sequence of measurable functions $\{f_n\}, f_n : \mathbb{R} \to \mathbb{R}$ for $n \geq 1$, which of the following are true? *1 point*

- If $f_n \to f$ in L^1 norm, then $f_n \to f$ pointwise almost everywhre.
- If $f_n \to f$ in measure, and $f_n \to g$ in L^1 norm, then $f = g$ a.e.
- If $f_n \to f$ in L^∞ norm, then there exists a subsequence f_{n_k} of $\{f_n\}$ such that f_{n_k} converges to f in L^1 norm.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $f_n \to f$ in measure, and $f_n \to g$ in L^1 norm, then $f = g$ a.e.

8) Let $A \subset \mathbb{R}^d$ be Lebesgue measurable. Then there exists a sequence of continuous functions $\{f_n\}$ such that $\{f_n\} \to \chi_A$ in measure. [Hint: Assume the following fact: For a metric space X and two disjoint nonempty closed subsets $A, B \subset X$, there exists a continuous function $f : X \to [0,1]$ such that $f|_A \equiv 0$ and $f|_B \equiv 1$. Consider $F_n \subset A \subset U_n$ with F_n closed and U_n open for $n \geq 1$ and $m(U_n \backslash F_n) < 1/n$. Now consider the closed sets F_n and $\mathbb{R}^d \backslash U_n$] *1 point*

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
True

9) Suppose $f : \mathbb{R}^d \to \mathbb{R}$ is measurable. Then there exists a sequence of simple measurable functions $\{\phi_n\}, \phi_n : \mathbb{R}^d \to \mathbb{R}$ such that $\{\phi_n\}$ converges to f in measure. [Hint:Consider $f : \mathbb{R}^2 \to \mathbb{R}, (x,y) \mapsto x$] *1 point*

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
False

10) Let $\{f_n\}, \{g_n\}$ be sequences of measurable functions $f_n, g_n : \mathbb{R}^d \to \mathbb{C}$ and $f, g : \mathbb{R}^d \to \mathbb{C}$ be measurable. Then which of the following statements are true? *1 point*

- If $f_n \to f, g_n \to g$ in measure, then $f_n + g_n \to f + g$ in measure.
- If $f_n \to f, g_n \to g$ in measure, then $f_n . g_n \to f . g$ in measure.
- If $f_n \to f, g_n \to g$ in L^∞ norm, then $\{f_n . g_n\} \to f . g$ in L^∞ norm

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $f_n \to f, g_n \to g$ in measure, then $f_n + g_n \to f + g$ in measure.

11) Choose the correct statements from the following, assuming that all sequences of functions appearing below consist of (real or complex) measurable functions. *1 point*

- Let $(X, \mathcal{B}, \mu)$ be a measure space. $\{f_n\}$ be a sequence of measurable complex valued functions on X. If $f_n \to f$ in L^∞ norm then $f_n \to f$ in measure.
- If μ is the counting measure on $\mathbb{R}$ then $f_n \to f$ in measure if and only if $f_n \to f$ uniformly everywhere.
- If μ is the Dirac delta measure on $\mathbb{R}$ at 0 then $f_n \to f$ in measure if and only if $f_n(0) \to f(0)$. Recall that for a nonempty set X and $x \in X$ the Dirac delta measure at x defined on $\mathcal{P}(X)$ denoted by δ_x is defined by

$$\delta_x(A) := \begin{cases} 1 & x \in A \\ 0 & otherwise \end{cases}$$

for $A \subset X$

No, the answer is incorrect.
Score: 0
Accepted Answers:
Let $(X, \mathcal{B}, \mu)$ be a measure space. $\{f_n\}$ be a sequence of measurable complex valued functions on X.
If $f_n \to f$ in L^∞ norm then $f_n \to f$ in measure.
If μ is the counting measure on $\mathbb{R}$ then $f_n \to f$ in measure if and only if $f_n \to f$ uniformly everywhere.
If μ is the Dirac delta measure on $\mathbb{R}$ at 0 then $f_n \to f$ in measure if and only if $f_n(0) \to f(0)$. Recall that for a nonempty set X and $x \in X$ the Dirac delta measure at x defined on $\mathcal{P}(X)$ denoted by δ_x is defined by

$$\delta_x(A) := \begin{cases} 1 & x \in A \\ 0 & otherwise \end{cases}$$

for $A \subset X$