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Course outline
                                                  Assignment 8
How does an NPTEL online
                                                  The due date for submitting this assignment has passed.
                                                                                                                                                                                       Due on 2020-11-11, 23:59 IST.
course work?
                                                  As per our records you have not submitted this assignment.
Week 0
                                                  1) Let f_n:\mathbb{R} 	o \mathbb{R} be defined as f_n(x) = sin(n)\chi_{[0,\frac{1}{n}]}(x), n \geq 1. Then which of the following are true?
                                                                                                                                                                                                                            1 point
Week 1
                                                    (f_n) 	o 0 in measure
Week 2
                                                   (f_n) 	o 0 almost uniformly
Week 3
                                                    (f_n) 	o 0 in L^1 norm
Week 4
Week 5
                                                   (f_n)	o 0 in L^\infty norm
                                                  No, the answer is incorrect.
Week 6
                                                  Score: 0
                                                  Accepted Answers:
Week 7
                                                  (f_n) 	o 0 in measure
                                                  (f_n) \to 0 almost uniformly
                                                  (f_n) 	o 0 in L^1 norm
Week 8
  Various modes of convergence
                                                 2) Let f_n:\mathbb{R}	o\mathbb{R} be defined as f_n=n\chi_{[1-\frac{1}{n},1+\frac{1}{n}]} , n\geq 1. Then which of the following are true?
                                                                                                                                                                                                                            1 point
   of measurable functions
   Easy implications from one
    mode of convergence to
                                                    (f_n) \to 0 pointwise everywhere.
    Implication map for modes of
                                                    (f<sub>n</sub>) → 0 pointwise almost everywhere.
    convergence with various
    examples
                                                    (f_n) 	o 0 uniformly a.e.
   Uniqueness of limits across
    various modes of convergence
                                                    (f_n) \to 0 uniformly everywhere.
    Some criteria for reverse
                                                  No, the answer is incorrect.
    implications for modes of
                                                  Score: 0
   convergence
                                                  Accepted Answers:
                                                  (f_n) \to 0 pointwise almost everywhere.
   Quiz : Assignment 8
   Week 8 Feedback Form :
                                                 Let f_n:\mathbb{R}\to\mathbb{R} be defined as f_n(x)=rac{1}{n}\chi_{[0,\infty)(x)}-rac{1}{n}\chi_{(-\infty,0]}(x)+\chi_{\{0\}}(x), n\geq 1. Then which of the following are true?
                                                                                                                                                                                                                            1 point
   Measure Theory
   Assignment 8 solutions
                                                   (f_n) \to 0 pointwise everywhere.
  Lecture notes
Week 9
                                                    (f_n) 	o 0 pointwise almost everywhere.
Week 10
                                                    (f_n) 	o 0 uniformly a.e.
Week 11
                                                   (f_n) \to 0 uniformly everywhere.
Week 12
                                                  No, the answer is incorrect.
                                                  Accepted Answers:
Video Download
                                                  (f_n) \to 0 pointwise almost everywhere.
                                                  (f_n) 	o 0 uniformly a.e.
Text Transcripts
                                                 Let f_n:\mathbb{R}	o\mathbb{R} be defined as f_n(x):=rac{sin(rac{1}{n})}{x^2+n^2} Let f=\sum_{n\geq 1}f_n . Then
                                                                                                                                                                                                                            1 point
Live Session
                                                   \sum_{n\geq 1}f_n converges absolutely and uniformly everywhere
                                                   The partial sums g_N := \sum_{n=1}^N f_n are L^1 functions for each N \geq 1
                                                   f\in L^1(\mathbb{R},m)
                                                   ||f-g_N||_{L^1}	o 0 as N	o \infty, where g_N is defined as above.
                                                  No, the answer is incorrect.
                                                  Score: 0
                                                  \sum_{n\geq 1} f_n converges absolutely and uniformly everywhere
                                                  The partial sums g_N := \sum_{n=1}^N f_n are L^1 functions for each N \geq 1
                                                  ||f-g_N||_{L^1}	o 0 as N	o \infty, where g_N is defined as above.
                                                 5) Let f_n:[0,1]	o \mathbb{R} be defined recursively as f_0(x)=x orall x \in [0,1]
                                                                                                                                                                                                                            1 point
                                                             f_n(x) = \left\{ egin{array}{ll} rac{1}{2} f_{n-1}(3x) & if \ x \in [0,1/3] \ rac{1}{2} & if \ x \in (1/3,2/3) \ rac{1}{2} + rac{1}{2} f_{n-1}(3x-2) & if \ x \in [2/3,1] \end{array} 
ight.
                                               [Hint: Draw the graphs of f_0, f_1, and f_2. Establish an inequality of the form |f_{n+1}-f_n|\leq \frac{1}{2}|f_n-f_{n-1}|, using induction. ]
                                                   (f_n) converges pointwise everywhere to a function f:[0,1]	o\mathbb{R}
                                                    (f_n) converges uniformly everywhere to a function f:[0,1]	o\mathbb{R}
                                                   (f_n) converges in L^1 norm to a function f:[0,1]	o\mathbb{R}
                                                  No, the answer is incorrect.
                                                  Score: 0
                                                  Accepted Answers:
                                                  (f_n) converges pointwise everywhere to a function f:[0,1]	o \mathbb{R}
                                                  (f_n) converges uniformly everywhere to a function f:[0,1] \to \mathbb{R}
                                                  (f_n) converges in L^1 norm to a function f:[0,1]	o\mathbb{R}
                                                  6) For a sequence of measurable functions \{f_n\}, f_n: \mathbb{R} \to \mathbb{R} for n \geq 1, which of the following are true?
                                                                                                                                                                                                                            1 point
                                                   If f_n 	o f in L^1 norm, then f_n 	o f in measure.
                                                   If f_n 	o f in measure, then f_n 	o f in L^1 norm
                                                   If f_n 	o f in measure, then f_n 	o f pointwise a.e.
                                                   If f_n 	o f in measure, then f_n 	o f almost uniformly.
                                                  No, the answer is incorrect.
                                                  Score: 0
                                                  Accepted Answers:
                                                  If f_n	o f in L^1 norm, then f_n	o f in measure.
                                                 7) For a sequence of measurable functions \{f_n\}, f_n: \mathbb{R} \to \mathbb{R} \ \ 	ext{for} \ \ n \geq 1, which of the following are true?
                                                                                                                                                                                                                            1 point
                                                   If f_n 	o f in L^1 norm, then f_n 	o f pointwise almost everywhre.
                                                   If f_n 	o f in measure, and f_n 	o g in L^1 norm, then f = g a.e.
                                                   If f_n	o f in L^\infty norm, then there exists a subsequence f_{n_k} of \left\{f_n
ight\} such that f_{n_k} converges to f in L^1 norm.
                                                  No, the answer is incorrect.
                                                  Accepted Answers:
                                                 If f_n 	o f in measure, and f_n 	o g in L^1 norm, then f = g a.e.
                                                 8) Let A \subset \mathbb{R}^d be Lebesgue measurable. Then there exists a sequence of continuous functions \{f_n\} such that \{f_n\} \to \chi_A in measure. [Hint: Assume 1 point
                                               the following fact: For a metric space X and two disjoint nonempty closed subsets A,B\subset X, there exists a continuous function f:X	o [0,1] such that
                                               f|_A\equiv 0 and f|_B\equiv 1. Consider F_n\subset A\subset U_n with F_n closed and U_n open for n\geq 1 and m(U_n\backslash F_n)<1/n. Now consider the closed sets F_n and \mathbb{R}^d\backslash U_n ]
                                                   True

    False

                                                  No, the answer is incorrect.
                                                  Accepted Answers:
                                                  True
                                                 9) Suppose f: \mathbb{R}^d \to \mathbb{R} is measurable. Then there exists a sequence of simple measurable functions \{\phi_n\}, \phi_n: \mathbb{R}^d \to \mathbb{R} such that \{\phi_n\} converges to 1 point
                                               f in measure. [ Hint:Consider f:\mathbb{R}^2	o\mathbb{R}, (x,y)\mapsto x ]
                                                    False
                                                  No, the answer is incorrect.
                                                  Score: 0
                                                  Accepted Answers:
                                                  False
                                                 10) Let \{f_n\}, \{g_n\} be sequences of measurable functions f_n, g_n : \mathbb{R}^d \to \mathbb{C} and f, g : \mathbb{R}^d \to \mathbb{C} be measurable. Then which of the following statements 1 point
                                               are true?
                                                   If f_n 	o f, g_n 	o g in measure, then f_n + g_n 	o f + g in measure.
                                                   If f_n 	o f, g_n 	o g in measure, then f_n \ldotp g_n 	o f \ldotp g in measure.
                                                   If f_n 	o f, g_n 	o g in L^\infty norm, then ig\{f_n,g_nig\} 	o f,g in L^\infty norm
                                                  No, the answer is incorrect.
                                                  Score: 0
                                                  Accepted Answers:
                                                 If f_n 	o f, g_n 	o g in measure, then f_n + g_n 	o f + g in measure.
                                                  11) Choose the correct statements from the following, assuming that all sequences of functions appearing below consist of (real or complex) measurable
                                               functions.
                                                   Let (X,\mathcal{B},\mu) be a measure space. \{f_n\} be a sequence of measurable complex valued functions on X. If f_n	o f in L^\infty norm then f_n	o f in measure.
                                                   If \mu is the counting measure on \mathbb R then f_n	o f in measure if and only if f_n	o f uniformly everywhere.
                                                   If \mu is the Dirac delta measure on \mathbb R at 0 then f_n \to f in measure if and only if f_n(0) \to f(0). Recall that for a nonempty set X and x \in X the Dirac delta
                                                   measure at x defined on \mathcal{P}(X) denoted by \delta_x is defined by
                                                            \delta_x(A) \coloneqq \left\{egin{array}{ll} 1 & x \in A \ 0 & otherwise \end{array}
ight.
                                                   forA\subset X
                                                  No, the answer is incorrect.
                                                 Let (X,\mathcal{B},\mu) be a measure space. \{f_n\} be a sequence of measurable complex valued functions on X.
                                                  If f_n 	o f in L^\infty norm then f_n 	o f in measure.
                                                 If \mu is the counting measure on \mathbb R then f_n	o f in measure if and only if f_n	o f uniformly everywhere.
                                                 If \mu is the Dirac delta measure on \mathbb R at 0 then f_n	o f in measure if and only if f_n(0)	o f(0). Recall that
                                                  for a nonempty set X and x \in X the Dirac delta measure at x defined on \mathcal{P}(X) denoted by \delta_x is defined
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for $A\subset X$