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NPTEL » Measure Theory
                                                                                                                                Announcements About the Course Ask a Question Progress Mentor
Unit 9 - Week 7
   Course outline
                                                        Assignment 7
   How does an NPTEL online
                                                        The due date for submitting this assignment has passed.
                                                                                                                                                                                                    Due on 2020-11-04, 23:59 IST.
   course work?
                                                        As per our records you have not submitted this assignment.
   Week 0
   Week 1
                                                       Let f:[a,b]	o\mathbb{R} be Riemann integrable. Let \left\{\phi_k
ight\} be a non-decreasing sequence of piecewise constant functions such that
                                                       \phi_k \leq f \, orall k \geq 1, |\phi_k(x)| \leq \sup_{[a,b]} |f(x)| orall x \in [a,b] and
   Week 2
                                                                       \int_a^b \phi_k(x) dx 	o \int_a^b f(x) dx, \;\; 	ext{as } k 	o \infty
   Week 3
   Week 4
                                                        (This sequence exists due to the Riemann-Darboux theorem). Similarly, let \{\psi_k\} be a sequence of non increasing piecewise constant functions such that
                                                        f \leq \psi_k, |\psi_k(x)| \leq \sup_{x \in [a,b]} |f(x)|, orall x \in [a,b] and
   Week 5
                                                                        \int_a^b \psi_k(x) dx 	o \int_a^b f(x) dx, as k 	o \infty
   Week 6
   Week 7
                                                        Now answer the next two questions.
      Lebesgue integral for complex
       and real measurable functions:
       the space of L^1 functions
                                                        1) Which of the following are true?
                                                                                                                                                                                                                                            1 point
       Basic properties of L^1-
       functions and Lebesgue's
      Dominated convergence
                                                          \phi = \lim_{k 	o \infty} \phi_k and \psi = \lim_{k 	o \infty} \psi_k are measurable functions
       theorem
      L^1 functions on R^d: Egorov's
                                                          \phi and \psi defined above are Lebesgue integrable on [a,b]
       theorem revisited (Littlewood's
       third principle)
                                                         The Riemann integral of \phi_k, \int_a^b \phi_k(x) dx is equal to the Lebesgue integral of \phi_k, \int_{[a,b]} \phi_k dm for each k \geq 1
      L^1 functions on R^d:
       Statement of Lusin's theorem
       (Littlewood's second principle),
       Density of simple functions,
                                                         The Riemann integral of \psi_k, \int_a^b \psi_k(x) dx is NOT equal to the Lebesgue integral of \psi_k, \int_{[a,b]} \psi_k dm for each k \geq 1
       step functions, and continuous
       compactly supported functions
       in L^1
                                                        No, the answer is incorrect.
                                                        Score: 0
      L^1 functions on R^d: Proof of
                                                        Accepted Answers:
      Lusin's theorem, space of L^1
                                                        \phi = \lim_{k 	o \infty} \phi_k and \psi = \lim_{k 	o \infty} \psi_k are measurable functions
      functions as a metric space
                                                        \phi and \psi defined above are Lebesgue integrable on [a,b]
    L^1 functions on R^d: the
                                                       The Riemann integral of \phi_k, \int_0^v \phi_k(x) dx is equal to the Lebesgue integral of \phi_k, \int_{(a,b)} \phi_k dm for
       Riesz-Fischer theorem
       Quiz : Assignment 7
                                                        each k \geq 1
      Week 7 Feedback Form :
                                                        2) Which of the following are true?
       Measure Theory
                                                                                                                                                                                                                                            1 point
      Assignment 7 solutions
                                                         \lim_{k	o\infty}\int_{[a,b]}\phi_k\,dm=\int_{[a,b]}\phi\,dm and \int_{[a,b]}\psi_k\,dm=\int_{[a,b]}\psi\,dm
   Week 8
   Week 9
                                                          f may not be a real measurable function
   Week 10
                                                          f=\phi=\psi for x\,a.\,e. in [a,b]
   Week 11
                                                         The Riemann integral f, \int_a^b f(x) dx is equal to the Lebesgue integral of f, \int_{[a,b]} f dm.
   Week 12
                                                        No, the answer is incorrect.
   Video Download
                                                       \lim_{k	o\infty}\int_{[a,b]}\phi_k\,dm=\int_{[a,b]}\phi\,dm and \int_{[a,b]}\psi_k\,dm=\int_{[a,b]}\psi\,dm f=\phi=\psi for x~a.~e. in [a,b]
   Text Transcripts
   Live Session
                                                       The Riemann integral f, \int_a^b f(x) dx is equal to the Lebesgue integral of f, \int_{[a,b]} f dm.
                                                       3) Let a\in\mathbb{R}. Let f_a:\mathbb{R}\to[0,\infty] be defined by f_a(x)=e^{-x}x^{a-1}\chi_{[0,\infty)}(x). Which of the following are true?
                                                                                                                                                                                                                                            1 point
                                                    [Hint: Use the fact that \int_{[0,\infty)} f_a \, dm = \lim_{n \to \infty} \int_{1/n}^1 f_a(x) dx + \lim_{n \to \infty} \int_1^n f_a(x) dx, where the integrals under the limits are ordinary Riemann integrals]
                                                         f_a \in L^1(\mathbb{R},m) for a>0
                                                          f_a \in L^1(\mathbb{R},m) for a>-1
                                                         f_a
ot\in L^1(\mathbb{R},m) for any a\in\mathbb{R}
                                                        No, the answer is incorrect.
                                                        Score: 0
                                                        Accepted Answers:
                                                        f_a \in L^1(\mathbb{R},m) for a>0
                                                        4) Let d\in\mathbb{N}, \sigma\in[0,\infty) and f:\mathbb{R}^d	o[0,\infty) be defined as
                                                                                                                                                                                                                                            1 point
                                                                       f(x) := (1 + ||x||^{\sigma})^{-1}
                                                     Choose the correct statements from the following.
                                                     Hint: To estimate \int_{\overline{B(0,1)}^c} f\,dm, Let A_k=ig\{x\in\overline{B(0,1)}^c: 2^k<||x||\le 2^{k+1}ig\}, k\ge 0 . Note that f(x)\le (1+2^{k\sigma})^{-1} on A_k . Define
                                                     lpha_k(x)=(1+2^{k\sigma})^{-1}\chi A_k(x), k\geq 0. Now f(x)\leq \sum_0^\infty lpha_k(x) and use Tonelli's theorem.
                                                          f \in L^1(\mathbb{R}^d,m) if and only if \,\sigma > d\,
                                                          f \in L^1(\mathbb{R}^d,m) if and only if 0 < \sigma < d
                                                         f\in L^1(\mathbb{R}^d,m) if \sigma=d
                                                         f \in L^1(\mathbb{R}^d,m) if d/2 \leq \sigma \leq d
                                                        No, the answer is incorrect.
                                                        Score: 0
                                                        Accepted Answers:
                                                        f \in L^1(\mathbb{R}^d,m) if and only if \,\sigma > d\,
                                                        In the following questions 5-8, use the following fact wherever applicable: the improper Riemann integral \int_0^\infty f(x)dx := \lim_{R \to \infty} \int_0^R f(x)dx coincides with the
                                                        Lebesgue integral for any unsigned measurable function f, which is continuous [0,\infty)
                                                        Evaluate the following limit.
                                                                                                                                                                                                                                            1 point
                                                              \lim_{n \to \infty} \int_0^\infty \left(1 + \frac{x}{n}\right)^{-n} \cos\left(\frac{x}{n}\right) dx
                                                     [Hint: Use the inequality: (1+x/n)^{-n} \leq (1+x+x^2/4)^{-1} for x\geq 0 ]
                                                          \bigcirc 0
                                                         01
                                                          \infty
                                                        No, the answer is incorrect.
                                                        Accepted Answers:
                                                        6) Evaluate the following limit:
                                                                                                                                                                                                                                            1 point
                                                               \lim_{n	o\infty}\int_0^1rac{ln(1+nx)}{(1+x)^n}dx
                                                     [Hint: Use the inequality (1+x)^n \geq 1+nx, for x \geq 0 ]
                                                         \bigcirc 0
                                                         01
                                                          \infty
                                                        No, the answer is incorrect.
                                                        Score: 0
                                                        Accepted Answers:
                                                        Evaluate the following limit:
                                                                                                                                                                                                                                            1 point
                                                                \lim_{n\to\infty}\int_0^\infty\int_0^n\sin(x)e^{-tx}dxdt
                                                     [Hint : Evaluate the inner integral explicitly.]
                                                          \pi/2
                                                          01
                                                        No, the answer is incorrect.
                                                        Score: 0
                                                        Accepted Answers:
                                                        8) Evaluate the limit:
                                                                                                                                                                                                                                           0 points
                                                                    \lim_{n	o\infty}\int_0^\infty n\,sinigg(rac{x}{n}igg)[x(1+x^2)^{-1}]dx
                                                     [Hint:Evaluate the limit \lim_{\epsilon \to 0^+} \frac{\sin \epsilon}{\epsilon}]
                                                          \pi/2
                                                          2\pi
                                                        No, the answer is incorrect.
                                                        Score: 0
                                                        Accepted Answers:
                                                        9) Let f \in L^1(\mathbb{R}^d,m). Which of the following are true?
                                                                                                                                                                                                                                            1 point
                                                          Given \,\epsilon>0, there exists R>0 such that \int_{B(0,R)^c}|f|dm<\epsilon
                                                          If f_n is a sequence of integrable functions such that ||f_n-f||_{L^1} 	o 0 as n 	o \infty then f_n converges to f pointwise almost everywhere.
                                                          For n \geq 1 let g_n = \min \big\{ |f|, n \big\} . Then each g_n is integrable and \lim_{n \to \infty} \int_{\mathbb{R}^d} g_n dm = \int_{\mathbb{R}^d} |f| \, dm
                                                         Let \{f_n\} be a sequence of integrable functions such that ||f_n-f||_{L^1}	o 0 as n	o\infty. Then given \epsilon>0 and \lambda\geq 1 there exists a N\in\mathbb{N} such that
                                                         mig(ig\{x\in\mathbb{R}^d:|f_n(x)-f(x)|>\lambdaig\}ig)\leq\epsilon\ orall n\geq N
                                                        Score: 0
                                                        Accepted Answers:
                                                       Given \,\epsilon>0, there exists R>0 such that \int_{B(0,R)^c}|f|dm<\epsilon
                                                        For n\geq 1 let g_n=\minig\{|f|,nig\} . Then each g_n is integrable and \lim_{n	o\infty}\int_{\mathbb{R}^d}\,g_ndm=\int_{\mathbb{R}^d}|f|\,dm
                                                       Let \{f_n\} be a sequence of integrable functions such that ||f_n-f||_{L^1}	o 0 as n	o \infty. Then given
                                                        \epsilon>0 and \lambda\geq 1 there exists a N\in\mathbb{N} such that mig(ig\{x\in\mathbb{R}^d:|f_n(x)-f(x)|>\lambdaig\}ig)\leq\epsilon\ orall n\geq N
                                                        10) Let f \in L^1(\mathbb{R}, m). Then, given \epsilon > 0 there exists \delta > 0 such that if A \subset \mathbb{R}^d is a measurable subset such that m(A) < \delta, then \int_A |f| dm < \epsilon
                                                         ○ True
                                                          False
                                                        No, the answer is incorrect.
                                                        Score: 0
                                                        Accepted Answers:
                                                        11) Suppose \{f_n\} is a sequence in L^1(\mathbb{R},m) and f:\mathbb{R}\to\mathbb{C} is a measurable fuction such that \int_\mathbb{R}|f_n-f|\to 0 as n\to\infty. Then which of the
                                                     following are true?
                                                         There exists M>0 such that |f_n(x)|\leq M \,\, orall n\geq 1 \,\, 	ext{for } a.\,e.\,x\in\mathbb{R}
                                                         f\in L^1(\mathbb{R},m)
                                                         There exists a subsequence \{f_{n_k}\} of \{f_n\} such that \{f_{n_k}\} converges point wise a.e. to f
                                                        No, the answer is incorrect.
                                                        Score: 0
                                                        Accepted Answers:
                                                        f\in L^1(\mathbb{R},m)
                                                        There exists a subsequence \{f_{n_k}\} of \{f_n\} such that \{f_{n_k}\} converges point wise a.e. to f
                                                        Choose the correct statements from the following.
                                                                                                                                                                                                                                            1 point
                                                         Let f \in L^1(\mathbb{R}^d, m), and let \epsilon > 0. Then there exists a Lebesgue measurable set E \subset \mathbb{R}^d, m(E) < \epsilon such that the restriction of f to \mathbb{R}^d \setminus E is continuous.
                                                         Let f \in L^1(\mathbb{R}^d, m), and let \epsilon > 0. Then there exists a Lebesgue measurable set E \subset \mathbb{R}^d, m(E) < \epsilon such that f is continuous on every point in \mathbb{R}^d \setminus E,
                                                          viewed as a function on \mathbb{R}^d
                                                         Let f \in L^1(\mathbb{R}^d,m) , then f is continuous on a G_\delta subset of \mathbb{R}^d
                                                        No, the answer is incorrect.
                                                        Let f\in L^1(\mathbb{R}^d,m), and let \epsilon>0. Then there exists a Lebesgue measurable set E\subset\mathbb{R}^d, m(E)<\epsilon
                                                        such that the restriction of f to \mathbb{R}^d \backslash E is continuous.
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13) Let (X, \mathcal{B}, μ) be a measure space and $f: X \to \mathbb{C}$ be measurable. Then which of the following are true?

If f_n is a sequence of integrable functions converging uniformly to f, then f is integrable.

If f_n is a sequence of bounded integrable functions converging uniformly to f, then f is integrable.

Suppose $\mu(X) < \infty$ and the sequence $ig\{f_nig\}$ converges to f pointwise a.e. to f . If $|f_n(x)| \leq M$ for

14) Let (X, \mathcal{B}, μ) be a measure space and $f: X \to \mathbb{C}$ be measurable. Then which of the following are true?

 $a.\,e.\,x\in X$ and for every $n\geq 1$, then f is integrable and $\lim_{n o\infty}\int_X f_n d\mu=\int_X f d\mu$

Suppose $\mu(X)<\infty$ and the sequence $\big\{f_n\big\}$ converges to f pointwise a.e. to f . If $|f_n(x)|\leq M$ for $a.e.\ x\in X$ and for every $n\geq 1$, then f is integrable

If $\mu(X) < \infty$, (a) holds.

No, the answer is incorrect.

If $\mu(X) < \infty$, (a) holds.

Accepted Answers:

Score: 0

and $\lim_{n o\infty}\int_X f_n d\mu = \int_X f d\mu$

If f is bounded a.e., then it is integrable.

If f is bounded a.e. and $\mu(X) < \infty$, then f is integrable.

If f is bounded a.e. and $\mu(X) < \infty$, then f is integrable.

The class of simple measurable functions on $\mathbb R$

The class of simple measurable functions on $\mathbb R$

The class of step functions on ${\mathbb R}$

No, the answer is incorrect.

The class of step functions on \mathbb{R}

Accepted Answers:

15) Which of the following are dense in the Banach space $L^1(\mathbb{R},m)$?

The class of compactly supported continuous functions on ${\mathbb R}$

The class of compactly supported continuous functions on $\mathbb R$

If $\mu(X) < \infty$ then f is integrable.

No, the answer is incorrect.

Accepted Answers:

Score: 0

Score: 0

1 point

1 point

1 point