Unit 8 - Week 6

Assignment 6

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-10-28, 23:59 IST.

- A statement on a measure space $(X, \mathcal{B}, \mu)$ is said to hold μ- almost everywhere (or simply almost everywhere when the underlying measure is clear from the context) if the statement holds on the complement of a μ null set. For example '$f : X \to \mathbb{R}$ is zero μ-almost everywhere' means that there exists $A \in \mathcal{B}$ and $\mu(A) = 0$ such that f is identically zero on A^c .
- Let $(X, \mathcal{B}, \mu)$ be a measure space and f be an unsigned measurable function on X. Then f is said to be integrable if and only if $\int_X f d\mu < \infty$

1) Let $(X, \mathcal{B})$ be a measurable space. Then which of the following statements are true? *1 point*

If f is $(\mathcal{B}, \mathcal{B}(\mathbb{R}^m))$ measurable, then f is $(\mathcal{B}, \mathcal{L}(\mathbb{R}^m))$ measurable.

If X is a totally ordered set with its order topology, then any order preserving $f : X \to [0, \infty]$ is measurable.

$\mathcal{A} = \{f : X \to \mathbb{C}$ is measurable$\}$ is a $\mathbb{C}$- vector space.

If $f : \mathbb{R} \to \mathbb{R}$ is measurable, then it is almost everywhere continuous. i.e, there exists $A \in \mathcal{L}(\mathbb{R})$ such that $\mu(A) = 0$ and f is continuous on A^c .

No, the answer is incorrect.
Score: 0
Accepted Answers:
If X is a totally ordered set with its order topology, then any order preserving $f : X \to [0, \infty]$ is measurable.
$\mathcal{A} = \{f : X \to \mathbb{C}$ is measurable$\}$ is a $\mathbb{C}$- vector space.

2) Let $(X, \mathcal{B}, \mu)$ be a measurable space.Let $f : X \to \mathbb{R}$ be measurable. Choose the correct statements: *1 point*

If $f \geq 0$ there exists a sequence (ϕ_n) of simple measurable functions such that $0 \leq \phi_1 \leq \phi_2 \leq \cdots \leq f$ and (ϕ_n) converges pointwise to f .

If $(X, \mathcal{B}, \mu)$ is σ- finite, there exists a sequence of simple measurable functions (ϕ_n) such that each (ϕ_n) is integrable and (ϕ_n) converges to f pointwise.

If f is bounded, $f \geq 0$, there exists a sequence (ϕ_n) of simple measurable functions such that $0 \leq \phi_1 \leq \phi_2 \leq \cdots \leq f$ and (ϕ_n) converges uniformly to f .

If $(X, \mathcal{B}, \mu)$ is σ- finite, there exists a sequence of simple measurable functions (ϕ_n) such that each (ϕ_n) is integrable and (ϕ_n) converges to f uniformly.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $f \geq 0$ there exists a sequence (ϕ_n) of simple measurable functions such that $0 \leq \phi_1 \leq \phi_2 \leq \cdots \leq f$ and (ϕ_n) converges pointwise to f .
If $(X, \mathcal{B}, \mu)$ is σ- finite, there exists a sequence of simple measurable functions (ϕ_n) such that each (ϕ_n) is integrable and (ϕ_n) converges to f pointwise.
If f is bounded, $f \geq 0$, there exists a sequence (ϕ_n) of simple measurable functions such that $0 \leq \phi_1 \leq \phi_2 \leq \cdots \leq f$ and (ϕ_n) converges uniformly to f .

3) Let $(X, \mathcal{B})$ be a measurable space. Choose the correct statements from the following: *1 point*

If $f, g : X \to \mathbb{C}$ are measurable and $g(x) \neq 0 \forall x \in X$, then f/g is measurable.

If $f, g : X \to \mathbb{C}$ are measurable , then $f.g$ is measurable.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $f, g : X \to \mathbb{C}$ are measurable and $g(x) \neq 0 \forall x \in X$, then f/g is measurable.
If $f, g : X \to \mathbb{C}$ are measurable , then $f.g$ is measurable.

4) For a measurable space $(X, \mathcal{B})$, which of the following are true? *1 point*

If $f_n : X \to \mathbb{R}$ is measurable $\forall n \in \mathbb{N}$ then $F : X \to \mathbb{R}$ defined by $F(x) = \sup_{n \to \infty} f_n(x)$ is measurable.

If $f_n : X \to \mathbb{R}$ is measurable $\forall n \in \mathbb{N}$ then $F : X \to \mathbb{R}$ defined by $F(x) = \limsup_{n \to \infty} f_n(x)$ is measurable.

If $f_n : X \to \mathbb{R}$ is measurable and (f_n) converges pointwise to $F : X \to \mathbb{R}$, then F is measurable.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $f_n : X \to \mathbb{R}$ is measurable $\forall n \in \mathbb{N}$ then $F : X \to \mathbb{R}$ defined by $F(x) = \sup_{n \to \infty} f_n(x)$ is measurable.
If $f_n : X \to \mathbb{R}$ is measurable $\forall n \in \mathbb{N}$ then $F : X \to \mathbb{R}$ defined by $F(x) = \limsup_{n \to \infty} f_n(x)$ is measurable.
If $f_n : X \to \mathbb{R}$ is measurable and (f_n) converges pointwise to $F : X \to \mathbb{R}$, then F is measurable.

5) For a measure space $(X, \mathcal{B}, \mu)$ which of the following statements are correct? *0 points*

If $f, g : X \to \mathbb{R}$ and $f = g \, \mu \, a.e.$ then g is measurable.

If $f_n : X \to \mathbb{R}$ and (f_n) converges to f pointwise $\mu \, a.e, .,.$ then f is measurable.

Both (a) and (b) are true if and only if X is a complete measure space. Recall that a measure space $(X, \mathcal{B}, \mu)$ is called complete if $A \in \mathcal{B}$ is such that $\mu(A) = 0$, then the power set of $A, \mathcal{P}(A) \subseteq \mathcal{B}$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Both (a) and (b) are true if and only if X is a complete measure space. Recall that a measure space $(X, \mathcal{B}, \mu)$ is called complete if $A \in \mathcal{B}$ is such that $\mu(A) = 0$, then the power set of $A, \mathcal{P}(A) \subseteq \mathcal{B}$.

6) Suppose $(X, \mathcal{B}, \mu)$ is a finite measure space and let $f_n : X \to [0, \infty]$ be measurable for $n \in \mathbb{N}$, and (f_n) converges pointwise to f , then f converges uniformly almost everywhere to f. *1 point*

True
False

No, the answer is incorrect.
Score: 0
Accepted Answers:
False

7) Which of the following are true? *1 point*

If $f : \mathbb{R}^d \to [0, \infty]$ is an unsigned measurable function and $g : \mathbb{R}^d \to \mathbb{C}$ is complex measurable, then the pointwise product of f and $Im(g)$ is a real measurable function.

If $f : \mathbb{R}^d \to \mathbb{C}$ is continuous, then it is measurable.

If $f : \mathbb{R}^d \to \mathbb{R}$ is continuous, and $E \subset \mathbb{R}$ is Lebesgue measurable, then $f^{-1}(E)$ is Lebesgue measurable.

The indicator function of a Lebesgue measurable subset of $\mathbb{R}^d$ is an unsigned measurable function.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $f : \mathbb{R}^d \to [0, \infty]$ is an unsigned measurable function and $g : \mathbb{R}^d \to \mathbb{C}$ is complex measurable, then the pointwise product of f and $Im(g)$ is a real measurable function.
If $f : \mathbb{R}^d \to \mathbb{C}$ is continuous, then it is measurable.
The indicator function of a Lebesgue measurable subset of $\mathbb{R}^d$ is an unsigned measurable function.

8) Let X be a set and $\mathcal{B}, \mathcal{B}'$ be two σ - algebras on X, let $Id : X \to X$ be the identity map, $x \mapsto x$. Then which of the following are true? *1 point*

If $\mathcal{B} \subseteq \mathcal{B}'$ then Id is $(\mathcal{B}, \mathcal{B}')$- measurable.

If $\mathcal{B}' \subseteq \mathcal{B}$ then Id is $(\mathcal{B}, \mathcal{B}')$- measurable.

Id is always $(\mathcal{B}, \mathcal{P}(X))$- measurable.

Id is always $(\mathcal{P}(X), \mathcal{P}(X))$- measurable.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $\mathcal{B}' \subseteq \mathcal{B}$ then Id is $(\mathcal{B}, \mathcal{B}')$- measurable.
Id is always $(\mathcal{P}(X), \mathcal{P}(X))$- measurable.

9) Let $f : \mathbb{R}^d \to \mathbb{R}$ and $g : \mathbb{R}^d \to \mathbb{R}^d$ be two functions. Then which of the following are true? *1 point*

If both f and g are continuous, $f \circ g$ is measurable.

If f is real measurable and g is continuous, then $f \circ g$ is measurable.

If f is continuous and g is $(\mathcal{B}(\mathbb{R}^d), \mathcal{B}(\mathbb{R}^d))$- measurable, then $f \circ g$ is measurable.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If both f and g are continuous, $f \circ g$ is measurable.
If f is continuous and g is $(\mathcal{B}(\mathbb{R}^d), \mathcal{B}(\mathbb{R}^d))$- measurable, then $f \circ g$ is measurable.

10) Let (f_n) be a sequence of real measurable functions on $\mathbb{R}^d$. Let $\lambda \in (0, \infty)$. Which of the following are true? *1 point*

$\{x \in \mathbb{R}^d : \sup_{n \in \mathbb{N}} f_n(x) > \lambda\} = \bigcup_{n \in \mathbb{N}} \{x \in \mathbb{R}^d : f_n(x) > \lambda\}$

$\{x \in \mathbb{R}^d : \sup_{n \in \mathbb{N}} f_n(x) > \lambda\} = \bigcap_{n \in \mathbb{N}} \{x \in \mathbb{R}^d : f_n(x) > \lambda\}$

$\{x \in \mathbb{R}^d : \inf_{n \in \mathbb{N}} \sup_{k \geq n} f_k(x) > \lambda\} = \bigcup_{M=1}^{\infty} \bigcap_{N=1}^{\infty} \{x \in \mathbb{R}^d : \sup_{k \geq N} f_k(x) > \lambda + 1/M\}$

$\{x \in \mathbb{R}^d : \inf_{n \in \mathbb{N}} \sup_{k \geq n} f_k(x) > \lambda\} = \bigcup_{M=1}^{\infty} \bigcap_{N=1}^{\infty} \bigcup_{k=N}^{\infty} \{x \in \mathbb{R}^d : f_k(x) > \lambda + 1/M\}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\{x \in \mathbb{R}^d : \sup_{n \in \mathbb{N}} f_n(x) > \lambda\} = \bigcup_{n \in \mathbb{N}} \{x \in \mathbb{R}^d : f_n(x) > \lambda\}$
$\{x \in \mathbb{R}^d : \inf_{n \in \mathbb{N}} \sup_{k \geq n} f_k(x) > \lambda\} = \bigcup_{M=1}^{\infty} \bigcap_{N=1}^{\infty} \{x \in \mathbb{R}^d : \sup_{k \geq N} f_k(x) > \lambda + 1/M\}$
$\{x \in \mathbb{R}^d : \inf_{n \in \mathbb{N}} \sup_{k \geq n} f_k(x) > \lambda\} = \bigcup_{M=1}^{\infty} \bigcap_{N=1}^{\infty} \bigcup_{k=N}^{\infty} \{x \in \mathbb{R}^d : f_k(x) > \lambda + 1/M\}$

11) Let $(X, \mathcal{B}, \mu)$ be a measure space such that $\mu(X) < \infty$. Suppose (f_n) is a sequence of functions, $f_n : X \to \mathbb{C}$ is a sequence of measurable functions such that (f_n) converges pointwise to $f : X \to \mathbb{C}$, then which of the following are true? *1 point*

If X is a topological space and f_n is continuous for each $n \geq 1$ then given $\epsilon > 0$ there exists a measurable subset $A_\epsilon \subseteq X$ such that $\mu(A_\epsilon) \leq \epsilon)$ and f is continuous when restricted to A_ϵ^c

If each f_n is integrable, then f is integrable.

There exists a measurable subset $A \subseteq X$ such that $\mu(A) = 0$ and f is continuous when restricted to A^c

No, the answer is incorrect.
Score: 0
Accepted Answers:
If X is a topological space and f_n is continuous for each $n \geq 1$ then given $\epsilon > 0$ there exists a measurable subset $A_\epsilon \subseteq X$ such that $\mu(A_\epsilon) \leq \epsilon)$ and f is continuous when restricted to A_ϵ^c

12) Let $s : \mathbb{R}^d \to [0, \infty)$ be a simple measurable function. Which of the following are true ? *1 point*

$\int_{\mathbb{R}^d} s \, dm = \sup\{ \int_{\mathbb{R}^d} \phi \, dm : \phi \leq s, \ \phi$ is simple measurable $\}$

$\int_{\mathbb{R}^d} s \, dm = \inf\{ \int_{\mathbb{R}^d} \psi \, dm : s \leq \psi, \ \psi$ is simple measurable $\}$

Let $r : \mathbb{R}^d \to [0, \infty)$ be another simple measurable function and $\alpha, \beta \in [0, \infty)$, then $\int_{\mathbb{R}^d} (\alpha s + \beta r) dm = \alpha \int_{\mathbb{R}^d} s \, dm + \beta \int_{\mathbb{R}^d} r \, dm$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\int_{\mathbb{R}^d} s \, dm = \sup\{ \int_{\mathbb{R}^d} \phi \, dm : \phi \leq s, \ \phi$ is simple measurable $\}$
$\int_{\mathbb{R}^d} s \, dm = \inf\{ \int_{\mathbb{R}^d} \psi \, dm : s \leq \psi, \ \psi$ is simple measurable $\}$
Let $r : \mathbb{R}^d \to [0, \infty)$ be another simple measurable function and $\alpha, \beta \in [0, \infty)$, then $\int_{\mathbb{R}^d} (\alpha s + \beta r) dm = \alpha \int_{\mathbb{R}^d} s \, dm + \beta \int_{\mathbb{R}^d} r \, dm$

13) Let (f_n) be a sequence of unsigned measurable function on a measure space $(X, \mathcal{B}, \mu)$ *1 point*

$_{n \to \infty} \int_X f_n \, d\mu$
$_{n \to \infty} f_n) d\mu = \limsup \int_X (\lim \sup$

$\int_X (\liminf_{n \to \infty} f_n) d\mu = \lim_{n \to \infty} \int_X (\inf_{k \geq n} f_k) \, d\mu$

$\int_X (\limsup_{n \to \infty} f_n) d\mu = \lim_{n \to \infty} \int_X (\sup_{k \geq n} f_k) \, d\mu$

$\int_X (\liminf_{n \to \infty} f_n) d\mu = \liminf_{n \to \infty} \int_X f_n \, d\mu$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\int_X (\liminf_{n \to \infty} f_n) d\mu = \lim_{n \to \infty} \int_X (\inf_{k \geq n} f_k) \, d\mu$

14) Suppose $(X, \mathcal{B}, \mu)$ is a measure space. Suppose $f : X \to \mathbb{R}$ is measurable. Which of the following are true? *1 point*

If f is non-negative and $\int_X f d\mu = 0$ then $f \equiv 0$

If $f_n : X \to [0, \infty]$ is measurable $\forall n \geq 0$, then $\int_X (\sum_{n=0}^{\infty} f_n) d\mu = \sum_{n=0}^{\infty} \int_X f_n \, d\mu$

Suppose $X = A \cup B$, $A, B \in \mathcal{B}$ with A, B disjoint. Let $\mathcal{C}_A = \{W \cap A : W \in \mathcal{B}\}$. Similarly define $\mathcal{C}_B$ .Let $\mu_A = \mu|_A$, similarly define μ_B . Now consider the measure spaces $(A, \mathcal{C}_A, \mu_A)$ and $(B, \mathcal{C}_B, \mu_B)$. Then $g : X \to \mathbb{C}$ is measurable if and only if $g|_A$ and $g|_B$ are measurable.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $f_n : X \to [0, \infty]$ is measurable $\forall n \geq 0$, then $\int_X (\sum_{n=0}^{\infty} f_n) d\mu = \sum_{n=0}^{\infty} \int_X f_n \, d\mu$
Suppose $X = A \cup B$, $A, B \in \mathcal{B}$ with A, B disjoint. Let $\mathcal{C}_A = \{W \cap A : W \in \mathcal{B}\}$. Similarly define $\mathcal{C}_B$ .Let $\mu_A = \mu|_A$, similarly define μ_B . Now consider the measure spaces $(A, \mathcal{C}_A, \mu_A)$ and $(B, \mathcal{C}_B, \mu_B)$. Then $g : X \to \mathbb{C}$ is measurable if and only if $g|_A$ and $g|_B$ are measurable.

15) Suppose $(X, \mathcal{B}, \mu)$ is a measure space. Suppose (f_n) is a sequence of unsigned integrable functions converging pointwise to f . If $f_n \geq f_{n+1} \forall n \geq 1$, then $\int_X f \, d\mu = \lim_{n \to \infty} \int_X f_n \, d\mu$ *1 point*

True
False

No, the answer is incorrect.
Score: 0
Accepted Answers:
False

16) Suppose $(X, \mathcal{B}, \mu)$ is a measure space. Suppose $f : X \to [0, \infty]$ are integrable. Then *1 point*

f^2 is integrable.

Let $A = \{x \in X : f(x) = \infty\}$.Then $\mu(A) = 0$

No, the answer is incorrect.
Score: 0
Accepted Answers:
Let $A = \{x \in X : f(x) = \infty\}$.Then $\mu(A) = 0$

17) Suppose $(X, \mathcal{B}, \mu)$ is a measure space.Suppose $f : X \to [0, \infty]$ is measurable.Define $\beta : \mathcal{B} \to [0, \infty]$ be defined by $\beta(A) = \int_A f \, d\mu$. *1 point*

β is a measure on $\mathcal{B}$.

If $\mu(A) = 0$ for some $A \in \mathcal{B}$, then $\beta(A) = 0$

$\mu(A) = 0$ for some $A \in \mathcal{B}$ if and only if $\beta(A) = 0$

$\beta(X) < \infty$ if and only if f is integrable.

No, the answer is incorrect.
Score: 0
Accepted Answers:
β is a measure on $\mathcal{B}$.
If $\mu(A) = 0$ for some $A \in \mathcal{B}$, then $\beta(A) = 0$
$\beta(X) < \infty$ if and only if f is integrable.