Unit 7 - Week 5

Course outline

Assignment 5

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-10-21, 23:59 IST.

1) Which of the following σ-algebras is/are equal to the Borel σ- algebra on $\mathbb{R}$? *1 point*

- The σ- algebra generated by compact subsets of $\mathbb{R}$.
- $\mathcal{A} := \{E : E \subset \mathbb{R}, E$ or E^c is countable$\}$
- The σ- algebra generated by $\mathcal{B} = \{(-a, a) : a \in \mathbb{R}\}$
- The σ algebra generated by finite and cofinite subsets of $\mathbb{R}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
The σ- algebra generated by compact subsets of $\mathbb{R}$.

2) Which of the following collection is a Boolean algebra? *0 points*

- The collection of all Lebesgue measurable subsets E of $\mathbb{R}^d$ such that E or E^c has finite Lebesgue measure.
- $\mathcal{F} = \{F : F \subset \mathbb{R}^d, m^*(F)$ or $m^*(F^c)$ is a positive integer$\}$
- The collection of all Lebesgue measurable subsets E of $\mathbb{R}^d$ such that E or E^c has zero Lebesgue measure.
- The collection of subsets E of $\mathbb{R}^d$ such that E or E^c is elementary.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The collection of all Lebesgue measurable subsets E of $\mathbb{R}^d$ such that E or E^c has finite Lebesgue measure.
$\mathcal{F} = \{F : F \subset \mathbb{R}^d, m^*(F)$ or $m^*(F^c)$ is a positive integer$\}$
The collection of subsets E of $\mathbb{R}^d$ such that E or E^c is elementary.

3) Which of the following collections form a Boolean algebra? *1 point*

- The collection of all subsets E of $\mathbb{R}^d$ such that E or E^c is a finite union of open Euclidean balls, or $E = \phi$.
- The collection of all subsets E of $\mathbb{R}^d$ such that E or E^c is finite.
- The collection of subsets E of $\mathbb{R}^d$ such that E or E^c is Jordan measurable.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The collection of all subsets E of $\mathbb{R}^d$ such that E or E^c is finite.
The collection of subsets E of $\mathbb{R}^d$ such that E or E^c is Jordan measurable.

4) Which of the following collection of subsets form a $\sigma-$ algebra? *1 point*

- The collection of Lebesgue measurable sets in $\mathbb{R}^d$
- The collection of Jordan measurable subsets of $[0, 1]$
- The collection $\mathcal{X} = \{f^{-1}(E) : E \subset \mathbb{R}\ is\ Borel\ measurable\ \}$ when $f : \mathbb{R}^d \to \mathbb{R}$ is continuous.
(A subset E of $\mathbb{R}$ is called Borel measurable if it belongs to the σ algebra generated by open subsets of $\mathbb{R}$.)
- The collection $\mathcal{C} = \{f(E) : E \subset \mathbb{R}^d\ is\ Lebesgue\ measurable\ \}$ and $f : \mathbb{R}^d \to \mathbb{R}$ is continuous.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The collection of Lebesgue measurable sets in $\mathbb{R}^d$
The collection $\mathcal{X} = \{f^{-1}(E) : E \subset \mathbb{R}\ is\ Borel\ measurable\ \}$ when $f : \mathbb{R}^d \to \mathbb{R}$ is continuous.
(A subset E of $\mathbb{R}$ is called Borel measurable if it belongs to the σ algebra generated by open subsets of $\mathbb{R}$.)

5) Assume the following fact: If $E \in \mathcal{B}(\mathbb{R}^2)$(Borel $\sigma-$ algebra) then $E_x := \{y \in \mathbb{R} : (x, y) \in E\}$ and $E_y := \{x \in \mathbb{R} : (x, y) \in E\}$ are Borel measurable subsets of $\mathbb{R}$, for any $x, y \in \mathbb{R}$. Choose the correct statements from the following: *1 point*

- Every Lebesgue measurable subset is Borel measurable.
- Every Borel measurable subset is Lebesgue measurable.
- Every Jordan null set is a Lebesgue null set.
- A bounded Lebesgue null set is Jordan null

No, the answer is incorrect.
Score: 0
Accepted Answers:
Every Borel measurable subset is Lebesgue measurable.
Every Jordan null set is a Lebesgue null set.

6) Let X be a non empty set. Suppose μ^* is an outer measure on X. Then if $A \subset X$ and $\mu^*(A) = 0$, then A is Caratheodory measurable. *1 point*

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
True

7) Let $\mathcal{B}$ be a Boolean algebra. Suppose μ_1 and μ_2 are two measures which are Hahn-Kolmogorov extensions of μ, restricted to the σ-algebra $\mathcal{A}$ generated by $\mathcal{B}$, such that $\mu_1 = \mu_2$ when restricted to $\mathcal{B}$. Then it holds that $\mu_1 = \mu_2$ when restricted to $\mathcal{A}$. [Hint: Let $X = (0, 1]$ and $\mathcal{B} = \{A \subset X : A \text{ is a finite union of intervals of the form } (a, b], 0 \leq a < b \leq 1\}$ and consider μ_1 to be the counting measure given by cardinality] *1 point*

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
False

8) Let $\mathcal{S}$ be the collection of all symmetric subsets of $\mathbb{R}$. (A subset B of $\mathbb{R}$ is called symmetric if $-a \in B$ whenever $a \in B$, or $B = \phi$). Then which of the following are true? *0 points*

- $\mathcal{S}$ is a $\sigma-$ algebra.
- $\mathcal{S}$ generates the discrete $\sigma-$ algebra (powerset).
- $\mathcal{S}$ is a Boolean algebra but not a $\sigma-$ algebra.
- $\mathcal{S}$ is a proper $\sigma-$ subalgebra of the Borel $\sigma-$ algebra.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mathcal{S}$ is a $\sigma-$ algebra.
$\mathcal{S}$ is a proper $\sigma-$ subalgebra of the Borel $\sigma-$ algebra.

9) Which of the following statements about The Lebesgue outer measure m^* are true? *1 point*

- $(\mathbb{R}^d, \mathcal{B}(\mathbb{R}^d), m^*)$ is a complete measure space.
- $(\mathbb{R}^d, \mathcal{L}(\mathbb{R}^d), m^*)$ is a complete measure space.
- $(\mathbb{R}^d, \overline{\mathcal{N}(\mathbb{R}^d)}, m^*)$ is a complete measure space.
- $(\mathbb{R}^d, \mathcal{C}_{m^*}(\mathbb{R}^d), m^*)$ is a complete measure space.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$(\mathbb{R}^d, \mathcal{L}(\mathbb{R}^d), m^*)$ is a complete measure space.
$(\mathbb{R}^d, \overline{\mathcal{N}(\mathbb{R}^d)}, m^*)$ is a complete measure space.
$(\mathbb{R}^d, \mathcal{C}_{m^*}(\mathbb{R}^d), m^*)$ is a complete measure space.

10) For the Lebesgue outer measure m^* on $\mathbb{R}^d$, which of the following are true? *1 point*

- $m^*(A) = \inf\{\Sigma_{n=1}^{\infty} m(A_i) : A \subseteq \bigcup_{n \in \mathbb{N}} A_i\ , A_i \in \mathcal{L}(\mathbb{R}^d)\}$
- $m^*(A) = \inf\{\Sigma_{n=1}^{\infty} m(A_i) : A \subseteq \bigcup_{n \in \mathbb{N}} A_i\ , A_i \in \mathcal{B}(\mathbb{R}^d)\}$
- $m^*(A) = \inf\{\Sigma_{n=1}^{\infty} m(A_i) : A \subseteq \bigcup_{n \in \mathbb{N}} A_i\ , A_i \in B,\ where\ B\ is\ a\ basis\ for\ topology\ on,\ \mathbb{R}^d\}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$m^*(A) = \inf\{\Sigma_{n=1}^{\infty} m(A_i) : A \subseteq \bigcup_{n \in \mathbb{N}} A_i\ , A_i \in \mathcal{L}(\mathbb{R}^d)\}$
$m^*(A) = \inf\{\Sigma_{n=1}^{\infty} m(A_i) : A \subseteq \bigcup_{n \in \mathbb{N}} A_i\ , A_i \in \mathcal{B}(\mathbb{R}^d)\}$
$m^*(A) = \inf\{\Sigma_{n=1}^{\infty} m(A_i) : A \subseteq \bigcup_{n \in \mathbb{N}} A_i\ , A_i \in B,\ where\ B\ is\ a\ basis\ for\ topology\ on,\ \mathbb{R}^d\}$

11) Let $\overline{\mathcal{E}(\mathbb{R}^d)}$ be the elementary algebra, i.e. the Boolean algebra generated by elementary subsets. Then which of the following statements about $< \overline{\mathcal{E}(\mathbb{R}^d)} >$ is true? $< \overline{\mathcal{E}(\mathbb{R}^d)} >$ is the $\sigma-$ algebra generated by $\overline{\mathcal{E}(\mathbb{R}^d)}$ *0 points*

- $< \overline{\mathcal{E}(\mathbb{R}^d)} >= \mathcal{B}(\mathbb{R}^d)$
- $< \overline{\mathcal{E}(\mathbb{R}^d)} >= \mathcal{L}(\mathbb{R}^d)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$< \overline{\mathcal{E}(\mathbb{R}^d)} >= \mathcal{L}(\mathbb{R}^d)$

12) Which of the following finitely additive measurs defined on the Elementary algebra on $\mathbb{R}$ are pre measures? *1 point*

- $m_0(I_{a,b}) = b - a$ where $I_{a,b}$ is any interval in $\mathbb{R}$ with end points a, b. $m_0(E) = \infty$ if E is co-elementary.
- $m_0(I_{a,b}) = \alpha(b) - \alpha(a)$ where $I_{a,b}$ is any interval in $\mathbb{R}$ with end points a, b. and $\alpha : \mathbb{R} \to \mathbb{R}$ is continuous, is non- decreasing and unbounded. $m_0(E) = \infty$ if E is co-elementary.
- $\exists c_0 \in R$ such that $m_0(I_{a,b}) = 1$ if $c_0 \in I_{a,b}$ and $m_0(I_{a,b}) = 0$ otherwise, where $I_{a,b}$ is any interval in $\mathbb{R}$ with end points a, b. $m_0(E) = \infty$ if E is co-elementary.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$m_0(I_{a,b}) = b - a$ where $I_{a,b}$ is any interval in $\mathbb{R}$ with end points a, b. $m_0(E) = \infty$ if E is co-elementary.
$m_0(I_{a,b}) = \alpha(b) - \alpha(a)$ where $I_{a,b}$ is any interval in $\mathbb{R}$ with end points a, b. and $\alpha : \mathbb{R} \to \mathbb{R}$ is continuous, is non- decreasing and unbounded.
$m_0(E) = \infty$ if E is co-elementary.

13) Let m_0 be the elementary measure on $\mathbb{R}^d$ extended as usual to $(\overline{\mathcal{E}(\mathbb{R}^d)}$. Which of the following is true ? *1 point*

- The Hahn Kolmogrov extension of $(\overline{\mathcal{E}(\mathbb{R}^d)}, m)$ is $\mathcal{L}(\mathbb{R}^d)$.
- The Hahn Kolmogrov extension of $(\mathcal{B}(\mathbb{R}^d), m)$ is $\mathcal{L}(\mathbb{R}^d)$.
- The Hahn Kolmogrov extension of $(\mathcal{L}(\mathbb{R}^d), m)$ is $\mathcal{L}(\mathbb{R}^d)$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The Hahn Kolmogrov extension of $(\overline{\mathcal{E}(\mathbb{R}^d)}, m)$ is $\mathcal{L}(\mathbb{R}^d)$.
The Hahn Kolmogrov extension of $(\mathcal{B}(\mathbb{R}^d), m)$ is $\mathcal{L}(\mathbb{R}^d)$.
The Hahn Kolmogrov extension of $(\mathcal{L}(\mathbb{R}^d), m)$ is $\mathcal{L}(\mathbb{R}^d)$.

14) Which of the following is true ? *1 point*

- If $\{E_i\}$ is a sequence of elementary subsets, then $\cup E_i$ can be co- elementary.
- If $\{E_i\}$ is a sequence of sets in $\mathcal{L}(\mathbb{R}^d)$, then there exists a sequence of pairwise disjoint subsets $\{B_i\}$ in $\mathcal{L}(\mathbb{R}^d)$ such that $\cup E_i = \cup B_i$.
- If $\{E_i\}$ is a sequence of Caratheodory measurable sets in X with respect to μ^*, then there exists a sequence of pairwise disjoint Caratheodory measurable sets $\{B_i\}$ such that $\cup E_i = \cup B_i$

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $\{E_i\}$ is a sequence of elementary subsets, then $\cup E_i$ can be co- elementary.
If $\{E_i\}$ is a sequence of sets in $\mathcal{L}(\mathbb{R}^d)$, then there exists a sequence of pairwise disjoint subsets $\{B_i\}$ in $\mathcal{L}(\mathbb{R}^d)$ such that $\cup E_i = \cup B_i$.
If $\{E_i\}$ is a sequence of Caratheodory measurable sets in X with respect to μ^*, then there exists a sequence of pairwise disjoint Caratheodory measurable sets $\{B_i\}$ such that $\cup E_i = \cup B_i$

15) Let μ^* be an outer measure on a set X. If E, F are Caratheodory measurable subsets of X, and $A \subset X$, consider the following subsets $B_i, i = 1, 2, \cdots, 5. B_1 = A \setminus (E \cup F), B_2 = (A \cap F) \setminus E, B_3 = (E \cap F) \setminus A, B_4 = (A \cap E) \setminus F, B_5 = E \cap F \cap A$. Then which of the following are true? *1 point*

- $\mu^*(A) = \mu^*(B_2 \cup B_4 \cup B_5) + \mu^*(B_1)$
- $\mu^*(B_1 \cup B_2) = \mu^*(B_2) + \mu^*(B_1)$
- $\mu^*(B_1 \cup B_4) = \mu^*(B_4) + \mu^*(B_1)$
- $\mu^*(A) = \mu^*(B_2 \cup B_4 \cup B_1) + \mu^*(B_5)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mu^*(A) = \mu^*(B_2 \cup B_4 \cup B_5) + \mu^*(B_1)$
$\mu^*(B_1 \cup B_2) = \mu^*(B_2) + \mu^*(B_1)$
$\mu^*(B_1 \cup B_4) = \mu^*(B_4) + \mu^*(B_1)$
$\mu^*(A) = \mu^*(B_2 \cup B_4 \cup B_1) + \mu^*(B_5)$

16) Let $F : \mathbb{R} \to \mathbb{R}, x \mapsto x^3$. Then F is strictly increasing and hence define its corresponding Lebesgue-Stieltjes pre-measure on the Boolean algebra generated by all intervals in $\mathbb{R}$ (both bounded and unbounded intervals), say $\overline{\mathcal{E}_F(\mathbb{R}^d)}$. Let μ_F be the Lebesgue- Stieltjes measure obtained by its Hahn-Kolmogrov extension. Which of the following are true? *1 point*

- $\mu_F((a, b)) = \mu_F((a, b]) = \mu_F([a, b)) = \mu_F([a, b]) \forall a, b \in \mathbb{R}$
- If C is the middle third Cantor set, then $\mu_F(C) = 0$.
- If $E \subset \mathbb{R}$ has finite Lebesgue measure, and if E is μ_F measurable, then $\mu_F(E) < \infty$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mu_F((a, b)) = \mu_F((a, b]) = \mu_F([a, b)) = \mu_F([a, b]) \forall a, b \in \mathbb{R}$
If C is the middle third Cantor set, then $\mu_F(C) = 0$.