NPTEL » Measure Theory Announcements About the Course Ask a Question Progress Mentor

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Unit 5 - Week 3
   Course outline
                                                   Assignment 3
   How does an NPTEL online
                                                    The due date for submitting this assignment has passed.
                                                                                                                                                                                      Due on 2020-10-07, 23:59 IST.
   course work?
                                                    As per our records you have not submitted this assignment.
   Week 0
                                                   Notation: We use the notation m_J(.) (respectively m^J(.)) for the inner Jordan measure (respectively the outer Jordan measure). If E \subset \mathbb{R}^d is elementary or
   Week 1
                                                    Jordan measurable, then the elementary or Jordan measure of E is denoted by m(E). The Lebesgue outer measure of a set E \subseteq \mathbb{R}^d is denoted as m^*(E). If
                                                    m^*(E) = 0, E will be called a Lebesgue null set.
   Week 2
   Week 3

    Let E ⊂ ℝ<sup>d</sup> be bounded. Which of the following statements are true?

                                                                                                                                                                                                                           1 point
      Outer measure - Motivation
       and Axioms of outer measure
      Comparing Inner Jordan
                                                      If E is Jordan measurable, then m^*(E) = m(E)
       measure, Lebesgue outer
       measure and Jordan Outer
                                                     m_J(E) \leq m^*(E)
       measure
      Finite additivity of outer
                                                     m_J(E) < m^*(E)
       measure on Separated sets,
      Outer regularity - Part 1
                                                      If E is open, then m^*(E) = m_J(E)
      Finite additivity of outer
                                                    No, the answer is incorrect.
       measure on Separated sets,
                                                    Score: 0
      Outer regularity - Part 2
                                                    Accepted Answers:
      Lebesgue measurable class of
                                                    If E is Jordan measurable, then m^*(E) = m(E)
       sets and their Properties - Part
                                                    m_J(E) \leq m^*(E)
                                                    If E is open, then m^*(E) = m_J(E)
      Lebesgue measurable class of
                                                   2) Let E \subset \mathbb{R}^d. Which of the following statements are true?
       sets and their Properties - Part
                                                                                                                                                                                                                           1 point
                                                        Every open set is Lebesgue measurable
      Equivalent criteria for lebesgue
                                                        Every closed set is Lebesgue measurable
       measurability of a subset - Part
                                                      If E is Jordan measurable, then E is Lebesgue measurable.
      Equivalent criteria for lebesgue
       measurability of a subset - Part
                                                      If E is Lebesgue measurable, then E is Jordan measurable.
                                                    No, the answer is incorrect.
      Quiz: Assignment 3
                                                    Score: 0
      Week 3 Feedback Form :
                                                    Accepted Answers:
      Measure Theory
                                                    Every open set is Lebesgue measurable
                                                    Every closed set is Lebesgue measurable
      Lecture Notes
                                                    If E is Jordan measurable, then E is Lebesgue measurable.
      Assignment 3 solution
                                                    3) For a sequence (a_n) of real numbers which of the following are true?
                                                                                                                                                                                                                          0 points
   Week 4
   Week 5
                                                      If E_n=(-\infty,a_n), n\geq 1 then \limsup_{n\to\infty}E_n=(-\infty,\limsup a_n)
   Week 6
                                                      If E_n=(a_n,\infty),\, n\geq 1 then \liminf_{n\to\infty}E_n=(\limsup\, a_n,\infty)
   Week 7
                                                      If E_n=(\min\big\{a_n,a_{n+1}\big\},\max\big\{a_n,a_{n+1}\big\}), n\geq 1 then \limsup_{n\to\infty}E_n is always non-empty.
   Week 8
                                                      If E_n=(\min\big\{a_n,a_{n+1}\big\},\max\big\{a_n,a_{n+1}\big\}), and the sequence \big\{a_n\big\} converges, then \limsup_{n\to\infty}E_n=\big\{\lim_{n\to\infty}a_n\big\}
   Week 9
                                                    No, the answer is incorrect.
                                                     Score: 0
                                                    Accepted Answers:
   Week 10
                                                    If E_n=(-\infty,a_n), n\geq 1 then \limsup_{n	o\infty}E_n=(-\infty,\limsup a_n)
                                                    If E_n=(a_n,\infty), n\geq 1 then \liminf_{n	o\infty}E_n=(\limsup \, a_n,\infty)
   Week 11
                                                    If E_n=(\min\{a_n,a_{n+1}\},\max\{a_n,a_{n+1}\}), and the sequence \{a_n\} converges, then
                                                   \limsup_{n \to \infty} \vec{E_n} = \left\{ \lim_{n \to \infty} a_n \right\}
   Week 12
                                                    4) Let 0 < a \le 1/3 and let C_a denote modified Cantor set corresponding to a. Which of the following are true?
   Video Download
                                                                                                                                                                                                                           1 point
   Text Transcripts
                                                      C_a is Jordan measurable if and only if a=1/3
   Live Session
                                                      C_a is Lebesgue measurable \forall a \in (0,1/3]
                                                      [0,1]ackslash C_a is Jordan measurable for some a such that 0 < a < 1/3
                                                      Let \{a_n\} be a sequence of real numbers such that 0 < a_n < 1/3, then \bigcap_{n > 1} C_{a_n} is Lebesgue measurable.
                                                    No, the answer is incorrect.
                                                    Accepted Answers:
                                                    C_a is Jordan measurable if and only if a=1/3
                                                    C_a is Lebesgue measurable \forall a \in (0,1/3]
                                                   Let \{a_n\} be a sequence of real numbers such that 0 < a_n < 1/3, then igcap_{n > 1} C_{a_n} is Lebesgue
                                                    measurable.
                                                   5) Let E \subset \mathbb{R}^d. Choose the statement which is NOT equivalent to other statements:
                                                                                                                                                                                                                           1 point
                                                      Given \epsilon>0 there exists an open set U such that m^*(U\,\Delta\,E)\leq\epsilon
                                                      Given \epsilon>0 there exists an open set U and a closed set F such that F\subseteq E\subseteq U and m^*(U\backslash F)\le \epsilon
                                                      Given \epsilon>0 there exists an elementary subset F\subseteq E such that m^*(E\backslash F)\leq \epsilon
                                                    No, the answer is incorrect.
                                                    Score: 0
                                                    Accepted Answers:
                                                    Given \epsilon>0 there exists an elementary subset F\subseteq E such that m^*(Eackslash F)\leq \epsilon
                                                   6) Let \{E_n\} be a collection of bounded Lebesgue measurable subsets of \mathbb{R}^d and let E=\bigcup_{n\geq 1}E_n. Which of the following are true, assuming that the 1 point
                                                 inequalities are considered in the extended non-negative reals [0, +\infty]?
                                                      Given \,\epsilon>0 , there exists a sequence of open sets \big\{U_n\big\} such that E_n\subseteq U_n and m^*(U_nackslash E_n)\leq \epsilon/2^n
                                                      There exists a sequence of compact sets \{K_n\} such that K_n\subseteq E_n and \lim_{n	o\infty}m^*(E_nackslash K_n)=0
                                                      The following inequality may hold: m^*(E) < m^*(\limsup_{n \to \infty} E_n)
                                                     \lim_{n \to \infty} m^*(E \backslash E_n) = 0
                                                    No, the answer is incorrect.
                                                    Score: 0
                                                    Accepted Answers:
                                                    Given \,\epsilon>0 , there exists a sequence of open sets \{U_n\} such that E_n\subseteq U_n and m^*(U_nackslash E_n)\leq \epsilon/2^n
                                                    There exists a sequence of compact sets \{K_n\} such that K_n\subseteq E_n and \lim_{n	o\infty}m^*(E_nackslash K_n)=0
                                                    7) Which of the following statements are true?
                                                                                                                                                                                                                           1 point
                                                      Every subset of the middle-thirds Cantor set is Lebesgue measurable.
                                                      Let E \subset \mathbb{R}^d and f: \mathbb{R}^d 	o \mathbb{R}^d be a homeomorphism. Then \, m^*(f(E)) = m^*(E) \,
                                                      Given \epsilon>0 there exists an open dense subset of \mathbb R of Lebesgue outer measure less than \epsilon.
                                                    No, the answer is incorrect.
                                                    Accepted Answers:
                                                    Every subset of the middle-thirds Cantor set is Lebesgue measurable.
                                                    Given \epsilon > 0 there exists an open dense subset of \mathbb R of Lebesgue outer measure less than \epsilon.
                                                    8) Which of the following statements are true?
                                                                                                                                                                                                                           1 point
                                                     If T:\mathbb{R}^d 	o \mathbb{R}^d is linear E is Lebesgue measurable, then m^*(T(E)) = m^*(E) is not always true.

    Lebesgue null sets are always bounded.

                                                      If A \subset \mathbb{R} has positive Lebesgue outer measure, then it contains a non-empty open set.
                                                    No, the answer is incorrect.
                                                   If T:\mathbb{R}^d	o\mathbb{R}^d is linear E is Lebesgue measurable, then m^*(T(E))=m^*(E) is not always true.
                                                   9) Let A\subset \mathbb{R}. Then A	imes \{0\} is Lebesgue measurable in \mathbb{R}^2
                                                                                                                                                                                                                           1 point
                                                     True

    False

                                                    No, the answer is incorrect.
                                                    Score: 0
                                                    Accepted Answers:
                                                    10) Which of the following options imply that a Lebesgue measurable set E\subset\mathbb{R}^d has finite Lebesgue outer measure?
                                                      There exists a compact set F such that m^*(E \Delta F) < \infty
                                                      E=igcap_{n>1}E_n where ig\{E_nig\} is a sequence of Lebesgue measurable sets such that \,m^*(E_n)<\infty\, for some \,n\in\mathbb{N}
                                                      E = \bigcup_{n \geq 1} E_n where \left\{E_n\right\} is a sequence of Lebesgue measurable sets such that m^*(E_n) < a_n for some sequence \left\{a_n\right\} such that \lim_{n \to \infty} a_n = 0.
                                                      E=igcup_{n\geq 1}E_n where \{E_n\} is a sequence of Lebesgue measurable sets such that m^*(E_n)< a_n for some sequence \{a_n\} such that \sum a_n<\infty
                                                    No, the answer is incorrect.
                                                    Score: 0
                                                    Accepted Answers:
                                                    There exists a compact set \,F\, such that \,m^*(E\,\Delta\,F)<\infty
                                                    E=igcap_{n\geq 1}E_n where ig\{E_nig\} is a sequence of Lebesgue measurable sets such that m^*(E_n)<\infty for
                                                    E = igcup_{n \geq 1} E_n where \{E_n\} is a sequence of Lebesgue measurable sets such that m^*(E_n) < a_n for
                                                   some sequence \left\{a_n
ight\} such that \sum a_n < \infty
                                                   Let E \subset \mathbb{R}^d be Lebesgue measurable. Then m^*(Int(E)) = m^*(E) = m^*(\overline{E}), where Int(E) is the interior of E and \overline{E} is the closure of E.
                                                     True

    False

                                                    No, the answer is incorrect.
                                                    Accepted Answers:
                                                    12) If E \subset \mathbb{R} is Lebesgue measurable, then its boundary \partial E is Lebesgue measurable and m^*(\partial E) = 0
                                                                                                                                                                                                                           1 point
                                                     ○ True
                                                     False
                                                    No, the answer is incorrect.
                                                    Accepted Answers.
                                                    False
                                                    13) Let f: \mathbb{R} \to \mathbb{R} be continuous. Then which of the following is true?
                                                                                                                                                                                                                           1 point
                                                      G_f = ig\{(x,f(x)): x \in \mathbb{R}ig\} is Lebesgue measurable in \mathbb{R}^2 and m(G_f) = 0
                                                      A_f \coloneqq ig\{(x,y) : 0 < y < f(x)ig\} is Lebesgue measurable
                                                      If E \subset \mathbb{R} has finite Lebesgue outer measure, then f(E) also has finite Lebesgue outer measure.
                                                     If E has positive Lebesgue outer measure, then f(E) has positive Lebesgue outer measure.
                                                    No, the answer is incorrect.
                                                    Accepted Answers:
                                                    G_f = ig\{(x,f(x)): x \in \mathbb{R}ig\} is Lebesgue measurable in \mathbb{R}^2 and m(G_f) = 0
                                                   A_f := ig\{ (x,y) : 0 < y < f(x) ig\} is Lebesgue measurable
                                                    14) Let C be the middle-third Cantor set. It can be proved that there exists a continuous surjection g:C\to [0,1]. Based on this fact, conclude
                                                                                                                                                                                                                           1 point
                                                 which of the following statements are true?
                                                      If E\subset\mathbb{R}^d is a Lebesgue null set and f:E	o\mathbb{R} is continuous (E having subspace topology from \mathbb{R} ), f(D) need not be a Lebesgue null set for each D\subset E.
                                                     If E \subset \mathbb{R}^d is a Lebesgue null set and f: E \to \mathbb{R} is continuous (E having subspace topology from \mathbb{R}), f(E) always has finite Lebesgue outer measure.
                                                     Let E,F\subset\mathbb{R}^d , if f:E	o F is uniformly continuous, then f preserves Lebesgue outer measure.
                                                    No, the answer is incorrect.
                                                    Score: 0
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Accepted Answers:

No, the answer is incorrect.

Accepted Answers:

f(D) need not be a Lebesgue null set for each $D \subset E$.

 $\mathcal{N}(\mathbb{R}^d)$ is closed under taking lim sup and lim inf

There exists an injective map from $\mathcal{N}(\mathbb{R}^d)$ into \mathbb{R}

 $\mathcal{N}(\mathbb{R}^d)$ is closed under taking lim sup and lim inf

if $A \in \mathcal{N}(\mathbb{R}^d)$ then its power set is a subset of $\mathcal{N}(\mathbb{R}^d)$.

if $A \in \mathcal{N}(\mathbb{R}^d)$ then its power set is a subset of $\mathcal{N}(\mathbb{R}^d)$.

If $E\subset\mathbb{R}^d$ is a Lebesgue null set and $f:E o\mathbb{R}$ is continuous (E having subspace topology from \mathbb{R}),

15) Let $\mathcal{N}(\mathbb{R}^d)$ be the collection of all Lebesgue null sets in \mathbb{R}^d . Then which of the following statements about $\mathcal{N}(\mathbb{R}^d)$ are true?

1 point