Unit 4 - Week 2

Assignment 2

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-09-30, 23:59 IST.

Notation: We use the notation $m_J(.)$ (respectively $m^J(.)$) for the inner Jordan measure (respectively the outer Jordan measure). If $E \subset \mathbb{R}^d$ is elementary or Jordan measurable, then the elementary or Jordan measure of E is denoted by $m(E)$(or $m^{(d)}(E)$ if the dimension needs to be specified.)

1) Which of the following statements are true? *1 point*

- The set $\mathbb{Q}^2 \cap [0,1]^2 = \{(x,y) \in [0,1]^2 : x, y \in Q\}$ is Jordan measurable.
- A disk in $\mathbb{R}^2$ is Jordan measurable.
- A solid sphere in $\mathbb{R}^3$ is Jordan measurable.
- A solid ellipsoid in $\mathbb{R}^3$ is Jordan measurable.

No, the answer is incorrect.
Score: 0
Accepted Answers:
A disk in $\mathbb{R}^2$ is Jordan measurable.
A solid sphere in $\mathbb{R}^3$ is Jordan measurable.
A solid ellipsoid in $\mathbb{R}^3$ is Jordan measurable.

2) Determine which of the following options is/are true. *0 points*

- If $E \subseteq \mathbb{R}$ is not a Jordan measurable subset then $E \times \{0\}$ may not be a Jordan measurable subset of $\mathbb{R}^2$
- A bounded set E in $\mathbb{R}^d$ is Jordan measurable if and only if ∂E is Jordan measurable.
- Every closed and bounded subset of $\mathbb{R}$ is Jordan measurable
- Every open and bounded subset of $\mathbb{R}$ is Jordan measurable

No, the answer is incorrect.
Score: 0
Accepted Answers:
A bounded set E in $\mathbb{R}^d$ is Jordan measurable if and only if ∂E is Jordan measurable.

3) Let $E \subset \mathbb{R}^d$ be a bounded set. Choose the statement which is NOT equivalent to the other statements: *1 point*

- E is Jordan measurable.
- Given $\epsilon > 0,$ there exists an elementary set $F_\epsilon \subset \mathbb{R}^d$ such that $m^J(E \,\Delta\, F_\epsilon) \leq \epsilon.$
- Given $\epsilon > 0,$ there exists an elementary set $F_\epsilon \subset \mathbb{R}^d$ such that $m_J(E \,\Delta\, F_\epsilon) \leq \epsilon.$
- Given $\epsilon > 0,$ there exists an elementary set $A \subseteq E$ and an elementary set $B \supseteq E$ such that $m(B \backslash A) \leq \epsilon.$

No, the answer is incorrect.
Score: 0
Accepted Answers:
Given $\epsilon > 0,$ there exists an elementary set $F_\epsilon \subset \mathbb{R}^d$ such that $m_J(E \,\Delta\, F_\epsilon) \leq \epsilon.$

4) Which of the following statements are true? *1 point*

- If $E \subset \mathbb{R}^d$ is Jordan measurable and $L : \mathbb{R}^d \to \mathbb{R}^d$ is a non-invertible linear transformation then $L(E) \subset \mathbb{R}^d$ may not be Jordan measurable.
- If $A \subset \mathbb{R}$ is Jordan measurable, then A^d is Jordan measurable in $\mathbb{R}^d$
- Middle-thirds Cantor set is Jordan measurable.
- Every countable bounded subset of $\mathbb{R}^n$ is Jordan measurable.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $A \subset \mathbb{R}$ is Jordan measurable, then A^d is Jordan measurable in $\mathbb{R}^d$
Middle-thirds Cantor set is Jordan measurable.

5) Choose the correct statements from the following: *1 point*

- There exists an uncountable Jordan measurable subset of [0, 1] which has Jordan measure 0.
- There exists a non-empty open Jordan measurable subset of [0, 1] which has Jordan measure 0.
- There exists a Jordan measurable subset of $\mathbb{R}^d$ which has infinite Jordan measure.
- Every $x \in \mathbb{R}^d$ has an open neighbourhood (depending on x) which is Jordan measurable, and has Jordan measure 10^{-5} .

No, the answer is incorrect.
Score: 0
Accepted Answers:
There exists an uncountable Jordan measurable subset of [0, 1] which has Jordan measure 0.
Every $x \in \mathbb{R}^d$ has an open neighbourhood (depending on x) which is Jordan measurable, and has Jordan measure 10^{-5} .

6) Let $\mathcal{E}(\mathbb{R}^d)$ be the collection of elementary subsets of $\mathbb{R}^d$ and $m' : \mathcal{E}(\mathbb{R}^d) \to \mathbb{R}^+$ be a set function, that obeys the properties of non-negativity, finite additivity and translation invariance. Suppose further that $m'([0,1]^d) = 2$. Determine which of the following are true: *1 point*

- $m'\left([0, \frac{1}{3}]^d\right) = \frac{2}{81},$ when $d = 4$
- $m'\left([0, \frac{1}{3}]^d\right) = \frac{1}{81},$ when $d = 4$
- $m'\left([0, \frac{1}{10}]^d\right) = \frac{1}{1000},$ when $d = 3$
- $m'\left([0, \frac{1}{10}]^d\right) = \frac{1}{100},$ when $d = 3$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$m'\left([0, \frac{1}{3}]^d\right) = \frac{2}{81},$ when $d = 4$

7) Which of the following statements is/ are true? *0 points*

- If A, B are Jordan measurable, then $A \backslash B$ is also Jordan measurable and $m(A \backslash B) = m(A) - m(B).$
- Countable union of Jordan measurable sets is Jordan measurable.
- For every bounded subset of $\mathbb{R}^d$, Jordan outer measure is well defined, and is finite.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If A, B are Jordan measurable, then $A \backslash B$ is also Jordan measurable and $m(A \backslash B) = m(A) - m(B).$
For every bounded subset of $\mathbb{R}^d$, Jordan outer measure is well defined, and is finite.

8) Let $A = \{(x,y) : 0 \leq x < 1, 0 < y < x^2\} \subseteq \mathbb{R}^2$. Then $m_J(A) = \cdots$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.3,0.35

1 point

9) Given a Jordan measurable subset E of $\mathbb{R}^d$, there exist a closed Jordan measurable subset E_1 and an open Jordan measurable subset E_2 such that $E_1 \subseteq E \subseteq E_2$ and $m_J(E_2 \backslash E_1) < 10^{-5}$. This statement is *1 point*

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
True

10) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by the matrix

$$\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$

Then for the unit square $A, m_J(T(A)) = \cdots$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 5.9,6.1

1 point

11) Let $f : [a,b] \to \mathbb{R}$ be continuous. Let $G(f) := \{(x, f(x)); x \in [a,b]\}$. Then $m_J(G(f)) = \cdots$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0,0.1

1 point

12) Let $E \subseteq \mathbb{R}^d$ be bounded. Let χ_E denote its characteristic function. Which of the given statements imply Jordan measurability of E? *0 points*

- χ_E has finitely many discontinuities
- χ_E is Riemann integrable
- χ_E is piecewise constant
- all of the these.

No, the answer is incorrect.
Score: 0
Accepted Answers:
all of the these.

13) For a set $E \subseteq \mathbb{R}^d$, denote the interior of E by $Int(E)$ and the closure of E by $\overline{E}$. If E is Jordan measurable, choose the correct statement(s) from the following: *1 point*

- $m_J(Int(E)) < m_J(E) < m_J(\overline{E})$
- $m_J(Int(E)) < m_J(E) = m_J(\overline{E})$
- $m_J(Int(E)) = m_J(E) = m_J(\overline{E})$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$m_J(Int(E)) = m_J(E) = m_J(\overline{E})$

14) Let $E \subseteq F$ be bounded subsets of $\mathbb{R}^d$. Which of the following statements are true? *1 point*

- $m_J(E) \leq m_J(F)$
- $m^J(E) \geq m^J(F)$
- $m^J(E) \leq m^J(F)$
- $m_J(E) \geq m_J(F)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$m_J(E) \leq m_J(F)$
$m^J(E) \leq m^J(F)$

15) Let E and F be two bounded disjoint subsets of $\mathbb{R}^d$. Which of the following statements are true? *1 point*

- $m_J(E \cup F) \geq m_J(E) + m_J(F)$
- $m_J(E \cup F) \leq m_J(E) + m_J(F)$
- $m_J(E \cup F) = m_J(E) + m_J(F)$
- Option (c) is true if F is an elementary subset of $\mathbb{R}^d$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$m_J(E \cup F) \geq m_J(E) + m_J(F)$
Option (c) is true if F is an elementary subset of $\mathbb{R}^d$.

16) Let $E \subset \mathbb{R}^d$ be bounded. Determine which of the following are true? *1 point*

- $m_J(E) = \sup_{A \subseteq E,\ A \text{ is a finite union of compact convex polytopes}} m(A)$
- $m_J(E) = \sup_{A \subseteq E,\ A \text{ is Jordan measurable}} m(A)$
- $m_J(E) = \sup_{A \subseteq E,\ A \text{ is finite}} |A|$, where $|A|$ is the cardinality of the finite set A

No, the answer is incorrect.
Score: 0
Accepted Answers:

$$m_J(E) = \sup_{A \subseteq E,\ A \text{ is a finite union of compact convex polytopes}} m(A)$$

$$m_J(E) = \sup_{A \subseteq E,\ A \text{ is Jordan measurable}} m(A)$$

17) A function $f : [a,b] \to [0,\infty)$ is said to be of bounded variation if the following quantity $V(f)$, known as the *total variation* of f , is finite: $V(f) := sup\{ \sum_{1=1}^{N} |f(x_{i+1}) - f(x_i)| : a = x_1 < x_2 < \cdots < x_{N+1} = b$ is a partition of $[a,b], N \in \mathbb{N}\}$ For a function $f : [a,b] \to [0,\infty)$ of bounded variation, determine which of the following are true: *1 point*

- f is Riemann integrable.
- f is continuous.
- The set $E_f \subseteq \mathbb{R}^2$ given by $E_f = \{(x,t) : x \in [a,b], 0 \leq t \leq f(x)\}$ is Jordan measurable.
- If $E \subseteq [a,b]$ and χ_E is of bounded variation on $[a,b]$, then E is Jordan measurable.

No, the answer is incorrect.
Score: 0
Accepted Answers:
f is Riemann integrable.
The set $E_f \subseteq \mathbb{R}^2$ given by $E_f = \{(x,t) : x \in [a,b], 0 \leq t \leq f(x)\}$ is Jordan measurable.
If $E \subseteq [a,b]$ and χ_E is of bounded variation on $[a,b]$, then E is Jordan measurable.

18) Let $\pi_i : \mathbb{R}^2 \to \mathbb{R}, i = 1,2$ be the projection onto the i -th coordinate, i.e. $\pi_i(x_1, x_2) = x_i$. Let $E \subseteq \mathbb{R}^2$ be bounded. Determine which of the following statements is true: *1 point*

- If $E \subseteq \mathbb{R}^2$ is Jordan measurable, then $\pi_i(E)$ is Jordan measurable for $i = 1, 2$.
- If $F \subset \mathbb{R}$ is Jordan measurable, then $\pi_1^{-1}(F) \cap \pi_2^{-1}(F)$ is Jordan measurable in $\mathbb{R}^2$
- If E is Jordan measurable and $\pi_i(E), i = 1,2$ are Jordan measurable then we must have $m^{(2)}(E) = m^{(1)}(\pi_1(E)).\, m^{(1)}(\pi_2(E)).$
- Option (c) holds if $E = E_1 \times E_2$ for Jordan measurable subsets E_1 and E_2

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $F \subset \mathbb{R}$ is Jordan measurable, then $\pi_1^{-1}(F) \cap \pi_2^{-1}(F)$ is Jordan measurable in $\mathbb{R}^2$
Option (c) holds if $E = E_1 \times E_2$ for Jordan measurable subsets E_1 and E_2