Unit 14 - Week 12

Assignment 12

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-12-09, 23:59 IST.

1) Which of the following are functions of bounded variation on $[0,1]$? *1 point*

- [ ] $f(x) := \begin{cases} x^{\frac{1}{2}}\sin(x^{-\frac{3}{2}}) & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$
- [ ] $f(x) := \begin{cases} x^{\frac{3}{2}}\sin(x^{-\frac{1}{2}}) & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$
- [ ] $f(x) := \begin{cases} 2^k & if\ \frac{k}{100} \leq x < \frac{k+1}{100}\ , k = 0, 1, \ldots 100 \\ 1 & if\ x = 1 \end{cases}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$f(x) := \begin{cases} x^{\frac{3}{2}}\sin(x^{-\frac{1}{2}}) & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$
$f(x) := \begin{cases} 2^k & if\ \frac{k}{100} \leq x < \frac{k+1}{100}\ , k = 0, 1, \ldots 100 \\ 1 & if\ x = 1 \end{cases}$

2) The following statement holds: *1 point*

A function $f : [a,b] \to \mathbb{R}$ is of bounded variation if and only if it is a difference of two monotonically increasing functions on $[a,b]$.

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
True

3) The following statement holds: Cantor-Lebesgue function (also known as the 'Devil's Staircase' function) is absolutely continuous on $[0,1]$. *1 point*

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
False

4) For a measurable function $f : [a,b] \to \mathbb{R}$ consider the Dini derivatives $\overline{D^+ f}, \underline{D^+ f}, \overline{D^- f},$ and $\underline{D^- f}$. Then which of the following are true? *1 point*

- [ ] $\underline{D^+ f} \leq \overline{D^+ f}$ everywhere.
- [ ] $\underline{D^- f} \geq \overline{D^- f}$ everywhere.
- [ ] If all Dini derivatives are finite and equal at $x_0 \in [a,b]$, then $f'(x_0)$ exixts.
- [ ] If f is continuous and is of bounded variation, then $f'(x)$ exists a.e.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\underline{D^+ f} \leq \overline{D^+ f}$ *everywhere.*
If all Dini derivatives are finite and equal at $x_0 \in [a,b]$, then $f'(x_0)$ exixts.
If f is continuous and is of bounded variation, then $f'(x)$ exists a.e.

5) Choose the correct statement from the following: *1 point*

- [ ] If $f : [a,b] \to \mathbb{R}$ of bounded variation, then it is a bounded function.
- [ ] Suppose $f : [a,b] \to \mathbb{R}$ is continuous, then it is of bounded variation.
- [ ] Suppose total variation of $f : [a,b] \to \mathbb{R}$ is zero, then f is constant.
- [ ] If $f, g : [a,b] \to \mathbb{R}$ is of bounded variation, then $f.g$ is of bounded variation

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $f : [a,b] \to \mathbb{R}$ of bounded variation, then it is a bounded function.
Suppose total variation of $f : [a,b] \to \mathbb{R}$ is zero, then f is constant.
If $f, g : [a,b] \to \mathbb{R}$ is of bounded variation, then $f.g$ is of bounded variation

6) Suppose $f_n : [a,b] \to \mathbb{R}$ is of bounded variation for each $n \geq 1$. If $\{f_n\}$ converges uniformly to $f : [a,b] \to \mathbb{R}$, then f is of bounded variation. *1 point*

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
False

7) Let $f : [a,b] \to \mathbb{R}$. Choose the correct statements from the following: *1 point*

- [ ] If f is continuous, f is absolutely continuous.
- [ ] If f is Lipschitz continuous, then f is absolutely continuous. (A function $g : [a,b] \to \mathbb{R}$ is said to be Lipschitz continuous if there exists $K > 0$ such that $|g(x) - g(y)| \leq K|x - y|$ for every $x, y \in [a,b]$.)
- [ ] If f is of bounded variation, then it is absolutely continuous.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If f is Lipschitz continuous, then f is absolutely continuous. (A function $g : [a,b] \to \mathbb{R}$ is said to be Lipschitz continuous if there exists $K > 0$ such that $|g(x) - g(y)| \leq K|x - y|$ for every $x, y \in [a,b]$.)

8) Choose the correct statements from the following: *1 point*

- [ ] If $f : [a,b] \to \mathbb{R}$ is absolutely continuous, then it is differentiable a.e. and f' is Lebesgue integrable.
- [ ] $f : [a,b] \to \mathbb{R}$ is absolutely continuous, then f is of the form $f(x) := \int_{[a,x]} g(y)dy + C$ for a L^1 function g on $[a,b]$ and $C \in \mathbb{R}$.
- [ ] If f is of the form $f(x) := \int_{[a,x]} g(y)dy$ for a L^1 function g on $[a,b]$,then f is absolutely continuous.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $f : [a,b] \to \mathbb{R}$ is absolutely continuous, then it is differentiable a.e. and f' is Lebesgue integrable.
$f : [a,b] \to \mathbb{R}$ is absolutely continuous, then f is of the form $f(x) := \int_{[a,x]} g(y)dy + C$ for a L^1 function g on $[a,b]$ and $C \in \mathbb{R}$.
If f is of the form $f(x) := \int_{[a,x]} g(y)dy$ for a L^1 function g on $[a,b]$,then f is absolutely continuous.