

## Unit 14 - Week 12

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## Assignment 12

The due date for submitting this assignment has passed.  
As per our records you have not submitted this assignment.

**Due on 2020-12-09, 23:59 IST.**

1) Which of the following are functions of bounded variation on  $[0, 1]$  ? 1 point

$$f(x) := \begin{cases} x^{\frac{1}{2}} \sin(x^{-\frac{3}{2}}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

$$f(x) := \begin{cases} x^{\frac{3}{2}} \sin(x^{-\frac{1}{2}}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

$$f(x) := \begin{cases} 2^k & \text{if } \frac{k}{100} \leq x < \frac{k+1}{100}, k = 0, 1, \dots, 100 \\ 1 & \text{if } x = 1 \end{cases}$$

No, the answer is incorrect.  
Score: 0

Accepted Answers:

$$f(x) := \begin{cases} x^{\frac{3}{2}} \sin(x^{-\frac{1}{2}}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

$$f(x) := \begin{cases} 2^k & \text{if } \frac{k}{100} \leq x < \frac{k+1}{100}, k = 0, 1, \dots, 100 \\ 1 & \text{if } x = 1 \end{cases}$$

2) The following statement holds: 1 point

A function  $f : [a, b] \rightarrow \mathbb{R}$  is of bounded variation if and only if it is a difference of two monotonically increasing functions on  $[a, b]$ .

 True

 False

No, the answer is incorrect.  
Score: 0

Accepted Answers:

True

3) The following statement holds: Cantor-Lebesgue function (also known as the 'Devil's Staircase' function) is absolutely continuous on  $[0, 1]$ . 1 point

 True

 False

No, the answer is incorrect.  
Score: 0

Accepted Answers:

False

4) For a measurable function  $f : [a, b] \rightarrow \mathbb{R}$  consider the Dini derivatives  $\overline{D^+}f, \underline{D^+}f, \overline{D^-}f$ , and  $\underline{D^-}f$ . Then which of the following are true? 1 point

$$\overline{D^+}f \leq \overline{D^+}f \text{ everywhere.}$$

$$\underline{D^-}f \geq \underline{D^-}f \text{ everywhere.}$$

If all Dini derivatives are finite and equal at  $x_0 \in [a, b]$ , then  $f'(x_0)$  exists.

If  $f$  is continuous and is of bounded variation, then  $f'(x)$  exists a.e.

No, the answer is incorrect.  
Score: 0

Accepted Answers:

$$\overline{D^+}f \leq \overline{D^+}f \text{ everywhere.}$$

If all Dini derivatives are finite and equal at  $x_0 \in [a, b]$ , then  $f'(x_0)$  exists.

If  $f$  is continuous and is of bounded variation, then  $f'(x)$  exists a.e.

5) Choose the correct statement from the following: 1 point

If  $f : [a, b] \rightarrow \mathbb{R}$  of bounded variation, then it is a bounded function.

Suppose  $f : [a, b] \rightarrow \mathbb{R}$  is continuous, then it is of bounded variation.

Suppose total variation of  $f : [a, b] \rightarrow \mathbb{R}$  is zero, then  $f$  is constant.

If  $f, g : [a, b] \rightarrow \mathbb{R}$  is of bounded variation, then  $f \cdot g$  is of bounded variation

No, the answer is incorrect.  
Score: 0

Accepted Answers:

If  $f : [a, b] \rightarrow \mathbb{R}$  of bounded variation, then it is a bounded function.

Suppose total variation of  $f : [a, b] \rightarrow \mathbb{R}$  is zero, then  $f$  is constant.

If  $f, g : [a, b] \rightarrow \mathbb{R}$  is of bounded variation, then  $f \cdot g$  is of bounded variation

6) Suppose  $f_n : [a, b] \rightarrow \mathbb{R}$  is of bounded variation for each  $n \geq 1$ . if  $\{f_n\}$  converges uniformly to  $f : [a, b] \rightarrow \mathbb{R}$ , then  $f$  is of bounded variation. 1 point

 True

 False

No, the answer is incorrect.  
Score: 0

Accepted Answers:

False

7) Let  $f : [a, b] \rightarrow \mathbb{R}$ . Choose the correct statements from the following: 1 point

If  $f$  is continuous,  $f$  is absolutely continuous.

If  $f$  is Lipschitz continuous, then  $f$  is absolutely continuous. (A function  $g : [a, b] \rightarrow \mathbb{R}$  is said to be Lipschitz continuous if there exists  $K > 0$  such that  $|g(x) - g(y)| \leq K|x - y|$  for every  $x, y \in [a, b]$ .)

If  $f$  is of bounded variation, then it is absolutely continuous.

No, the answer is incorrect.  
Score: 0

Accepted Answers:

If  $f$  is Lipschitz continuous, then  $f$  is absolutely continuous. (A function  $g : [a, b] \rightarrow \mathbb{R}$  is said to be Lipschitz continuous if there exists  $K > 0$  such that  $|g(x) - g(y)| \leq K|x - y|$  for every  $x, y \in [a, b]$ .)

8) Choose the correct statements from the following: 1 point

If  $f : [a, b] \rightarrow \mathbb{R}$  is absolutely continuous, then it is differentiable a.e. and  $f'$  is Lebesgue integrable.

If  $f : [a, b] \rightarrow \mathbb{R}$  is absolutely continuous, then  $f$  is of the form  $f(x) := \int_{[a,x]} g(y)dy + C$  for a  $L^1$  function  $g$  on  $[a, b]$  and  $C \in \mathbb{R}$ .

If  $f$  is of the form  $f(x) := \int_{[a,x]} g(y)dy$  for a  $L^1$  function  $g$  on  $[a, b]$ , then  $f$  is absolutely continuous.

No, the answer is incorrect.  
Score: 0

Accepted Answers:

If  $f : [a, b] \rightarrow \mathbb{R}$  is absolutely continuous, then it is differentiable a.e. and  $f'$  is Lebesgue integrable.

If  $f : [a, b] \rightarrow \mathbb{R}$  is absolutely continuous, then  $f$  is of the form  $f(x) := \int_{[a,x]} g(y)dy + C$  for a  $L^1$  function  $g$  on  $[a, b]$  and  $C \in \mathbb{R}$ .

If  $f$  is of the form  $f(x) := \int_{[a,x]} g(y)dy$  for a  $L^1$  function  $g$  on  $[a, b]$ , then  $f$  is absolutely continuous.