## Unit 13 - Week 11

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## Assignment 11 Due on 2020-12-02, 23:59 IST. The due date for submitting this assignment has passed. As per our records you have not submitted this assignment. **Notation:** For a measurable function $f:\mathbb{R}^d o\mathbb{C}$ , we denote by Mf the associated Hardy-Littlewood maximal function $Mf(x) \coloneqq sup_{r>0}rac{1}{m(B(x,r))}\int_{B(x,r)}|f|dm$ Definition:[Lebesgue point of density] Let $E\subset\mathbb{R}^d$ be Lebesgue measurable.We call a point $x\in\mathbb{R}^d$ a Lebesgue point of density for E if $\lim_{r o 0^+} rac{m(E \cap B(x,r))}{m(B(x,r))} = 1$ . Answer the following two questions based on this definition. 1) Let $E \subset \mathbb{R}^d$ be a Lebesgue measurable set of strictly positive measure. Then which of the following are true? [Hint:First assume that $m(E) < \infty$ and 1 point apply Lebesgue's differentiation theorem.] Almost every point of E is a Lebesgue point of density for EIt may hold that no point of E is a Lebesgue point of density for EAlmost every point of $E^c$ is not a Lebesgue point of density for ENo, the answer is incorrect. Score: 0 Accepted Answers: Almost every point of $oldsymbol{E}$ is a Lebesgue point of density for $oldsymbol{E}$ Almost every point of $oldsymbol{E}^c$ is not a Lebesgue point of density for $oldsymbol{E}$ 2) Let $E \subset \mathbb{R}^d$ be Lebesgue measurable. If there exist no points of density for E, then E is a Lebesgue null set. 1 point True False No, the answer is incorrect. Score: 0 Accepted Answers: True 3) Let $f\in L^1(\mathbb{R}^d,m)$ . Then the value of $\lim_{n o\infty} mig(ig\{x\in\mathbb{R}^d:Mf(x)\geq n^2ig\}ig)$ is 1 point $\bigcirc$ 0 01 $\infty$ No, the answer is incorrect. Accepted Answers: 4) Let $\{f_n\}$ be a sequence of measurable functions $f_n:\mathbb{R}^d\to\mathbb{C}$ and let $f:\mathbb{R}^d\to\mathbb{C}$ be measurable such that $\sum_{n=1}^\infty ||f-f_n||_{L^1}$ converges. 1 point Then the following holds: $M(f-f_n) o 0$ as $n o \infty$ pointwise a.e. True False No, the answer is incorrect. Score: 0 Accepted Answers: 5) Let $f\in L^1(\mathbb{R}^d).$ For $x\in\mathbb{R}$ define $G_x:[0,\infty) o\mathbb{C}$ by 1 point $G_x(r) := \left\{ egin{array}{ll} rac{1}{m(B(x,r))} \int_{B(x,r)} \ f \, dm & if \, r > 0 \ f(x) & if \, r = 0 \end{array} ight.$ Then $G_x$ is continuous on $[0,\infty)$ for almost every $x\in\mathbb{R}^d$ For almost every $x \in \mathbb{R}^d$ , $G_x$ is discontinuous at 0. If f is bounded, $G_x$ is also bounded. No, the answer is incorrect. Score: 0 Accepted Answers: $G_x$ is continuous on $[0,\infty)$ for almost every $x\in\mathbb{R}^d$ If f is bounded, $G_x$ is also bounded. Let $f \in L^1(\mathbb{R}^d)$ . Define the Lebesgue set $L_f$ of f by $L_f := \left\{x \in \mathbb{R}^d : f(x) < \infty \text{ and } \lim_{r \to 0^+} \frac{1}{m(B(x,r))} \int_{B(x,r)} |f(y) - f(x)| dy = 0 \right\}$ . Then $f \in L^1(\mathbb{R}^d)$ . $\mu(L_f)=0$ Almost every $x \in \mathbb{R}^d$ is in $L_f$ . $L_f$ is Lebesgue measurable. No, the answer is incorrect. Accepted Answers: Almost every $x \in \mathbb{R}^d$ is in $L_f$ . $L_f$ is Lebesgue measurable. 7) Let $f\in L^1(\mathbb{R}^d)$ and let $x\in\mathbb{R}^d$ be a point of continuity of f . If $L_f$ is defined as in previous question, then $x\in L_f$ . 1 point True False No, the answer is incorrect. Score: 0 Accepted Answers: