Unit 3 - Week 1

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Assignment 1

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-09-30, 23:59 IST.

Basic Set theory and Elementary subsets of $\mathbb{R}^d$

1) For a non empty set A, which of the following statements imply that the set is infinite? *1 point*

- There exists an injective map from A into a proper subset of A
- There exists a surjective map from A onto a proper subset of A
- There exists an injective map from $\mathbb{N}$ into A.

No, the answer is incorrect.
Score: 0
Accepted Answers:
There exists an injective map from A into a proper subset of A
There exists an injective map from $\mathbb{N}$ into A.

2) A set is finite if and only if it has finitely many subsets. *1 point*
This statement is:

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
True

3) Consider the following equivalence relation Δ on $\mathbb{R}:(a,b)\in\Delta=\{(a,b)\in\mathbb{R}:a-b\in\mathbb{Q}\}$ (or $a\sim b$ if and only if $a-b\in\mathbb{Q}$). *1 point*
Then, **we can construct** a subset B of $\mathbb{R}$ which contains exactly one element from each of the equivalence classes defined by Δ on $\mathbb{R}$. Given above statement, which of the following options are correct :

- If Axiom of choice is assumed, then above statement is true.
- If Axiom of choice is assumed, then above statement is false.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If Axiom of choice is assumed, then above statement is true.

4) Choose the correct statement(s) from the following: *1 point*

- If A, B are infinite sets, then $|A|=|B|$
- Given any non empty set $A,$ there exists a set B such that $|A|<|B|$.
- Suppose A, B be non-empty sets. If A is equinumerous with a subset of B and B is equinumerous with a subset of A, then $|A|=|B|$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Given any non empty set $A,$ there exists a set B such that $|A|<|B|$.
Suppose A, B be non-empty sets. If A is equinumerous with a subset of B and B is equinumerous with a subset of A, then $|A|=|B|$.

5) Define the 'measure' μ of a subset of $\mathbb{R}^3$ as follows: *1 point*

For $E\subseteq\mathbb{R}^3, \mu(E)=1$ if $(1,0,0)\in E, \mu(E)=0$ otherwise.

Which of the following statements about μ is/are true?

- μ is translation invariant
- μ is invariant under reflections.
- μ is finitely additive.
- μ is invariant under continuous maps.

No, the answer is incorrect.
Score: 0
Accepted Answers:
μ is finitely additive.

6) For a set X, denote its powerset by $\mathcal{P}(X)$. Which of the following pairs of sets are equinumerous? *1 point*

- $\mathbb{N},\mathbb{Q}$
- $\mathbb{Z},\mathbb{Q}^d$, where d is a positive integer greater than or equal to 1.
- $\mathbb{N},\mathcal{P}(\mathbb{N})$
- $\mathcal{P}(\mathbb{Z}),\mathcal{P}(\mathcal{P}(\mathbb{Z}))$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mathbb{N},\mathbb{Q}$
$\mathbb{Z},\mathbb{Q}^d$, where d is a positive integer greater than or equal to 1.

7) Which of the following map defines an injective function $f:\mathbb{Q}\to\mathbb{N}$, given $gcd(p,q)=1$ *1 point*

- $f\left(\frac{p}{q}\right)=p+q$
- $f\left(\frac{p}{q}\right)=2^p\times 3^q$
- $f\left(\frac{p}{q}\right)=p^2\times q^3$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$f\left(\frac{p}{q}\right)=2^p\times 3^q$

8) Which of the following statements are true for elementary subsets, E, F, K in $\mathbb{R}^d$, not necessarily disjoint *1 point*
(Hint: Make a Venn diagram and use additive properties of elementary measure):

- $m(E\cup F)=m(E)+m(F)$
- $m(E\cup F)=m(E)+m(F)-m(E\cap F)$
- $m(E\cup F\cup K)=m(E)+m(F)+m(K)-m(E\cap F\cap K)$
- $m(E\cup F\cup K)=m(E)+m(F)+m(K)-m(E\cap F)-m(F\cap K)-m(K\cap E)+m(E\cap F\cap K)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$m(E\cup F)=m(E)+m(F)-m(E\cap F)$
$m(E\cup F\cup K)=m(E)+m(F)+m(K)-m(E\cap F)-m(F\cap K)-m(K\cap E)+m(E\cap F\cap K)$

9) Let $\mathcal{E}(\mathbb{R}^d)$ be the collection of all elementary subsets of $\mathbb{R}^d$. Let $m:\mathcal{E}(\mathbb{R}^d)\to\mathbb{R}_{\geq 0}$ be the elementary measure. Which of the following statements is/are true? *1 point*

- Suppose $A,B\in\mathcal{E}(\mathbb{R}^d)$. Let $d(A,B):=inf\{d(a,b):a\in A,b\in B\}$. If $d(A,B)=0$, the $m(A\cup B)<m(A)+m(B)$.
- For two boxes A, B in $\mathbb{R}^d$, the set $A+B$ is a box, and $m(A+B)=m(A)+m(B)$, where $A+B=\{a+b:a\in A,b\in B\}$
- If $\alpha\in\mathbb{R}$ and $A\in\mathcal{E}(\mathbb{R}^d)$, then $m(\alpha A)=|\alpha|^d m(A)$, where $\alpha A=\{\alpha a:a\in A\}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $\alpha\in\mathbb{R}$ and $A\in\mathcal{E}(\mathbb{R}^d)$, then $m(\alpha A)=|\alpha|^d m(A)$, where $\alpha A=\{\alpha a:a\in A\}$

10) Which of the following statements about $\mathcal{E}(\mathbb{R}^d)$ is true? *1 point*

- $\mathcal{E}(\mathbb{R}^d)$ is a group under the binary operation $\triangle$ where $A \triangle B$ denotes symmetric difference of A and B.
- $\mathcal{E}(\mathbb{R}^d)$ is closed under complements.
- Union of a countably infinite ascending chain of elements in $\mathcal{E}(\mathbb{R}^d)$ lies in $\mathcal{E}(\mathbb{R}^d)$, i.e, if $E_1\subseteq E_2\subseteq\cdots$ is an infinite sequence in $\mathcal{E}(\mathbb{R}^d)$, then we must have $\cup_{n=1}^{\infty}E_n\in\mathcal{E}(\mathbb{R}^d)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mathcal{E}(\mathbb{R}^d)$ is a group under the binary operation $\triangle$ where $A \triangle B$ denotes symmetric difference of A and B.

11) For a bounded subset E of $\mathbb{R}^d$, define $\mu(E)=\lim_{N\to\infty}\frac{1}{N^d}\#(E\cap\frac{1}{N}\mathbb{Z}^d)$ where $\frac{1}{N}\mathbb{Z}^d:=\left\{\left(\frac{k_1}{N},\frac{k_2}{N},\ldots,\frac{k_d}{N}\right):k_i\in\mathbb{Z},\ for\ i=1,2,\ldots,d\right\}$ and $\# A$ denotes the cardinality of a set A. Then for $d=2,$ what is $\mu([0,2]^2)$? *1 point*

- ∞
- 0
- 4

No, the answer is incorrect.
Score: 0
Accepted Answers:
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