Unit 2 - Week 0

Course outline

Assignment 0

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-09-21, 23:59 IST.

Note : This assignment is only for practice purpose and it will not be counted towards the Final score

1) Let $\{a_n\}_{n\in\mathbb{N}}$ be a recursively defined sequence, such that $a_n = \sqrt{2a_{n-1}}$,and given $0 < a_1 < \sqrt{2}$: *1 point*

- a_n is a monotonic and bounded sequence
- a_n converges to 0
- a_n converges to 2
- a_n is bounded but not monotonic

No, the answer is incorrect.
Score: 0
Accepted Answers:
a_n is a monotonic and bounded sequence
a_n converges to 2

2) Let $\{a_n\}_{n\in\mathbb{N}}$ be a recursively defined sequence, such that *1 point*

$$a_n = \frac{3 + a_{n-1}}{1 + a_{n-1}}$$

and given $a_1 > \sqrt{3}$:

- $\{a_n\}$ is a monotonic and bounded sequence
- $\lim\limits_{n\to\infty}$ sup (a_n) is $\sqrt{3}$
- $\{a_n\}$ converges to $\sqrt{3}$
- $\{a_n\}$ is bounded but not monotonic

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\lim\limits_{n\to\infty}$ sup (a_n) is $\sqrt{3}$
$\{a_n\}$ converges to $\sqrt{3}$
$\{a_n\}$ is bounded but not monotonic

3) State which of the following statements are True? *1 point*

- $\exists$ a continuous function $f : \mathbb{R} \to \mathbb{R}$ with exactly n points of continuity
- $\exists$ a monotonic function $f : \mathbb{R} \to \mathbb{R}$ with uncountably many discontinuities
- A sequence $(a_n) \subseteq \mathbb{R}$ is convergent if and only if $\lim\limits_{n\to\infty}$ sup$(a_n) = \lim\limits_{n\to\infty}$ inf (a_n)

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\exists$ a continuous function $f : \mathbb{R} \to \mathbb{R}$ with exactly n points of continuity
A sequence $(a_n) \subseteq \mathbb{R}$ is convergent if and only if $\lim\limits_{n\to\infty}$ sup$(a_n) = \lim\limits_{n\to\infty}$ inf (a_n)

4) Which of the following are true? *1 point*

- $\sum_{n=1}^{n=\infty} \frac{3^{1-2n}}{n^2+1}$ converges
- $\sum_{n=1}^{n=\infty} \frac{1}{n\,ln(n)}$ converges

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\sum_{n=1}^{n=\infty} \frac{3^{1-2n}}{n^2+1}$ converges

5) Suppose $a, b > 0$. The value of $\lim\limits_{x\to 0}(ln(1-e^{-ax})) - (ln(1-e^{-bx}))$ = ____________ *1 point*

- 0
- The limit does not exist.
- ln (b/a)
- ln (a/b)

No, the answer is incorrect.
Score: 0
Accepted Answers:
ln (a/b)

6) Determine which of the following choices are true: *1 point*

- There exists a homeomorphism between (0, 1) and [0, 1].
- There exists no continuous bijection from [0, 1] onto (0, 1)
- Every continuous real-valued function on a compact subset of $\mathbb{R}$ is uniformly continuous
- If $f : [0,1] \to [0,1]$ is a continuous bijection then f is a homeomorphism.

No, the answer is incorrect.
Score: 0
Accepted Answers:
There exists no continuous bijection from [0, 1] onto (0, 1)
Every continuous real-valued function on a compact subset of $\mathbb{R}$ is uniformly continuous
If $f : [0,1] \to [0,1]$ is a continuous bijection then f is a homeomorphism.

7) Determine which of the following statements are true: *1 point*

- $f : [a,b] \to [a,b]$ is a continuous function, then f has a fixed point.
- Let $F \subseteq \mathbb{R}$ be a closed subset. Define for $x \in \mathbb{R}, d(x,F) := inf_{y\in F}|x-y|$, then the function $g(x) := d(x,F)$ is a continuous function on $\mathbb{R}$.
- A continuous function defined on a compact set $A \subseteq \mathbb{R}$ has a maxima and minima and attains them on A.
- if K, F are respectively compact and closed disjoint subsets of $\mathbb{R}$ then $d(K,F) := inf_{y\in K}d(x,F)$ is strictly positive.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$f : [a,b] \to [a,b]$ is a continuous function, then f has a fixed point.
Let $F \subseteq \mathbb{R}$ be a closed subset. Define for $x \in \mathbb{R}, d(x,F) := inf_{y\in F}|x-y|$, then the function $g(x) := d(x,F)$ is a continuous function on $\mathbb{R}$.
A continuous function defined on a compact set $A \subseteq \mathbb{R}$ has a maxima and minima and attains them on A.
if K, F are respectively compact and closed disjoint subsets of $\mathbb{R}$ then $d(K,F) := inf_{y\in K}d(x,F)$ is strictly positive.

8) Determine which of the following statements are true, regarding the uniform and pointwise convergence of the given sequence of functions $f_n : \mathbb{R} \to \mathbb{R}, n \geq 1$ given by $f_n(x) = x^n$? *1 point*

- $f_n(x)$ converges uniformly on (0, 1)
- $f_n(x)$ converge uniformly on [0, 1]
- $f_n(x)$ converges pointwise on [0, 1]
- $f_n(x)$ converges uniformly on $(0,\alpha)$, $\forall \alpha < 1$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$f_n(x)$ converges pointwise on [0, 1]
$f_n(x)$ converges uniformly on $(0,\alpha)$, $\forall \alpha < 1$

9) Given $\sum_{n=1}^{\infty} |a_n| < \infty$ then $\sum_{n=1}^{\infty} a_n\, sin\, nx$ converges uniformly as a series on all of $\mathbb{R}$ *1 point*

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
True

10) Consider a function $f : \mathbb{R} \to \mathbb{R}$. Determine which of the following are true, given $f_n(x) = f(x + \frac{1}{n}), n \geq 1$: *1 point*

- If $f(x)$ is continuous on $\mathbb{R}$, $f_n(x)$ converges to $f(x)$ uniformly on all of $\mathbb{R}$
- If f is continuous on $\mathbb{R}$, $f_n(x)$ converges pointwise to $f(x)$ on all of $\mathbb{R}$
- If f is uniformly continuous on $\mathbb{R}$, $f_n(x)$ converges uniformly to $f(x)$ on all of $\mathbb{R}$
- If f is continuous on $\mathbb{R}$, then on any given compact subset of $\mathbb{R}$, $f_n(x)$ converges to f uniformly.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If f is continuous on $\mathbb{R}$, $f_n(x)$ converges pointwise to $f(x)$ on all of $\mathbb{R}$
If f is uniformly continuous on $\mathbb{R}$, $f_n(x)$ converges uniformly to $f(x)$ on all of $\mathbb{R}$
If f is continuous on $\mathbb{R}$, then on any given compact subset of $\mathbb{R}$, $f_n(x)$ converges to f uniformly.

11) $F : [-1,1] \to \mathbb{R}, F(x) = \int_{-1}^{x} |t|dt$ is a C^2 function. *1 point*

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
False

12) Suppose f_n, f are real valued functions on [0, 1] such that $\{f_n\}$ converges to f pointwise. If each f_n is Riemann integrable, which of the following statements are true? *1 point*

- f must be Riemann integrable, and limit and integral can be exchanged i.e. $\int_0^1 f dx = \lim\limits_{n\to\infty} \int_0^1 f_n dx.$
- If the convergence of f_n to f is uniform, then f is Riemann integrable and limit and integral can be exchanged.
- f may be Riemann integrable, but it need not be true that $\int_0^1 f dx = \lim\limits_{n\to\infty} \int_0^1 f_n dx.$

No, the answer is incorrect.
Score: 0
Accepted Answers:
If the convergence of f_n to f is uniform, then f is Riemann integrable and limit and integral can be exchanged.
f may be Riemann integrable, but it need not be true that $\int_0^1 f dx = \lim\limits_{n\to\infty} \int_0^1 f_n dx.$

13) Consider a sequence of differentiable functions $f_n : [a,b] \to \mathbb{R}$ such that f_n' is continuous for each $n \geq 1$, and f_n converges pointwise to a differentiable function f . Which of the following options implies $f' = \lim\limits_{n\to\infty} f_n'$? *1 point*

- Convergence is uniform ie, $\{f_n\}$ converges uniformly to f .
- $\{f_n'\}$ converges uniformly to some function g.
- f' is continuous.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\{f_n'\}$ converges uniformly to some function g.

14) Which of the following real-valued functions on [0, 1] is not necessarily Riemann integrable? *1 point*

- a differentiable function
- a monotone function
- a characteristic function of a countable set

No, the answer is incorrect.
Score: 0
Accepted Answers:
a characteristic function of a countable set

15) For a continuous function $f : [0,1] \to \mathbb{C},$ the quantity $\lim\limits_{x\to 0+} \frac{1}{x}\int_0^x f(t)dt$: *1 point*

- Need not exist always.
- Exists only if f is differentiable on [0, 1] and equals $f'(0)$.
- Always exists and equals $f(0)$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Always exists and equals $f(0)$.

16) Given an open subset U of $\mathbb{R}^n$, there exists another open subset $V \subset U$such that $\overline{V} \subseteq U$ and $\overline{V}$ is compact. *1 point*

- True
- false

No, the answer is incorrect.
Score: 0
Accepted Answers:
True

17) Determine whether the following statement is true or false: There exists a countably infinite connected subset of $\mathbb{R}$. *1 point*

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
False

18) Determine whether the following statement is true or false: *1 point*

There does not exist a continuous function $f : \mathbb{R} \to [0,1]$ such that $f \equiv 1$ on [−0.99, 0.99] but $f \equiv 0$ outside [−1, 1].

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
False

19) Which of the following statements is/ are true? *1 point*

- Every continuous function $f : \mathbb{R} \to \mathbb{R}$ maps Cauchy sequences to Cauchy sequences.
- A function $f : \mathbb{R} \to \mathbb{R}$ which maps Cauchy sequences to Cauchy sequences must be continuous.
- If $f : [a,b] \to \mathbb{R}$ is differentiable, f' has intermediate value property.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Every continuous function $f : \mathbb{R} \to \mathbb{R}$ maps Cauchy sequences to Cauchy sequences.
If $f : [a,b] \to \mathbb{R}$ is differentiable, f' has intermediate value property.

20) Which of the following statements about the middle-thirds Cantor set is true? *1 point*

- It is nowhere dense.
- It is uncountable.
- It is countable.

No, the answer is incorrect.
Score: 0
Accepted Answers:
It is nowhere dense.
It is uncountable.

21) The value of $\lim\limits_{n\to\infty} \sum_{k=1}^{n} \frac{k}{n^2}$ is : *1 point*

- $1/2$
- 1
- The limit does not exist.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$1/2$

22) Choose the correct statement(s) about a sequence of real numbers $\{x_n\}$ from the following: *1 point*

- If every subsequence of $\{x_n\}$ converges to a real number a then the original sequence itself converges to a.
- If $\{x_n\}$ has a convergent subsequence, then $\{x_n\} converges.$
- There are bounded sequences $\{x_n\}$ which do not have any convergent subsequences.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If every subsequence of $\{x_n\}$ converges to a real number a then the original sequence itself converges to a.

23) Which of the following statement(s) is true? *1 point*

- $\sum_{n=1}^{\infty} \frac{sin\, n}{n^2}$ is absolutely convergent.
- $\sum_{n=1}^{\infty} \frac{n!}{3^n}$ is divergent.
- $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ converges.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\sum_{n=1}^{\infty} \frac{sin\, n}{n^2}$ is absolutely convergent.
$\sum_{n=1}^{\infty} \frac{n!}{3^n}$ is divergent.

24) Radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{5^n \times x^n}{(n+1)}$ is _____

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.1,0.3

1 point

25) Every rearrangement of an absolutely convergent series gives the same sum. *1 point*

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
True