NPTEL COURSE - Introduction to Commutative Algebra

Assignment - Week 9

(1) Let A be a subring of a ring B such that the set $B \setminus A$ is closed under multiplication. Show that A is integrally closed in B.
(2) Let A be a commutative ring and M be a Noetherian A-module. Prove that $A/\operatorname{Ann}(M)$ is a Noetherian ring.
(3) If A is Noetherian, then prove that any surjective homomorphism $\phi : A \to A$ is an isomorphism.
(4) Find a composition series for $\mathbb{Z}/30\mathbb{Z}$.