

NPTTEL COURSE - Introduction to Commutative Algebra

Assignment - Week 8

- (1) Let  $A$  be a subring of  $B$  such that  $B$  is integral over  $A$ . Let  $\mathfrak{n}$  be a maximal ideal of  $A$ . Is  $B_{\mathfrak{n}}$  necessarily integral over  $A_{\mathfrak{m}}$ ?
- (2) Let  $B$  be an integral extension of  $A$ . Prove that
  - (a) If  $x \in A$  is a unit in  $B$ , then it is a unit in  $A$ .
  - (b) the Jacobson radical of the Jacobson radical of  $A$  is the contraction of the Jacobson radical of  $B$ .
- (3) If  $A$  is an integral domain, then prove that  $A$  is integrally closed in  $A[x]$ . Give an example of  $A$  such that  $A$  is not integrally closed in  $A[x]$ .