NPTEL COURSE - Introduction to Commutative Algebra

Assignment - Week 7

(1) Suppose that for each prime ideal $\mathfrak{p} \subset A$, the local ring $A_{\mathfrak{p}}$ has no nonzero nilpotent elements. Prove that A has no nonzero nilpotent elements.
(2) Let I be an ideal and let $S = 1 + I = \{1 + x \ : \ x \in I\}$. Prove that S is a multiplicatively closed subset. Prove that $S^{-1}I$ is contained in the Jacobson radical of $S^{-1}A$.
(3) For two ideals I, J in A, prove that $I \subset J$ if and only if $I_{\mathfrak{m}} \subset J_{\mathfrak{m}}$ in $A_{\mathfrak{m}}$ for all maximal ideal $\mathfrak{m}$.
(4) Is $\sqrt{2+\sqrt{2}} + \frac{1}{2}\sqrt[3]{3} \in \mathbb{R}$ integral over $\mathbb{Z}$? Justify your answer.