

NPTEL COURSE - Introduction to Commutative Algebra

Assignment - Week 6

- (1) Prove that $\mathbb{Z}/n\mathbb{Z} \otimes_{\mathbb{Z}} \mathbb{Z}/m\mathbb{Z} \cong \mathbb{Z}/d\mathbb{Z}$, where $d = \gcd(m, n)$.
- (2) Let A be a local ring, M and N are finitely generated A -modules. Prove that if $M \otimes_A N = 0$, then either $M = 0$ or $N = 0$.
- (3) A multiplicatively closed subset S of A is said to be *saturated* if $xy \in S \Leftrightarrow x \in S$ and $y \in S$. Prove that S is saturated if and only if $A \setminus S$ is a union of prime ideals.
- (4) Prove that if S is a multiplicatively closed subset of A , then $S^{-1}A$ is a flat A -module.