

NPTEL COURSE - Introduction to Commutative Algebra

Assignment - Week 5

- (1) Let $M_1, M_2 \subset M$ be A -submodules of a given module M . Prove that if $M_1 + M_2$ and $M_1 \cap M_2$ are finitely generated, then so are M_1 and M_2 .
- (2) Let A be a UFD and $x, y \in A$ be such that $\gcd(x, y) = 1$. Let $I = (x, y) \subset A$. Prove that the sequence $0 \rightarrow A \xrightarrow{\phi} A^2 \xrightarrow{\psi} I \rightarrow 0$ is exact, where $\phi(a) = (-ya, xa)$ and $\psi((a, b)) = ax + by$.
- (3) Let $0 \rightarrow V_1 \rightarrow \cdots \rightarrow V_n \rightarrow 0$ be an exact sequence of finite dimensional vector spaces over a field k . Prove that $\sum_{i=1}^n (-1)^i \dim_k V_i = 0$.
- (4) Prove that $M \otimes N \cong N \otimes M$.