

NPTEL COURSE - Introduction to Commutative Algebra

Assignment - Week 3

- (1) Let I be an ideal in a commutative ring with identity, A , and P_1, \dots, P_r be prime ideals of A . Prove that if $I \subseteq \cup_{i=1}^r P_i$, then $I \subseteq P_i$ for some $1 \leq i \leq r$.
- (2) Find the nilradical of \mathbb{Z}_{36} and \mathbb{Z}_9 .
- (3) Let F be a field and let $R = F[x, y]$. If $I = (x^2, xy)$ and $S = \{x\}$, then compute $I : S$.
- (4) If M is an A -module, then prove that $\text{Hom}_A(A, M) \cong M$.