

NPTEL COURSE - Introduction to Commutative Algebra

Assignment - Week 2

- (1) Let S be a set and $\mathcal{F}(S) := \{f : S \rightarrow \mathbb{R} \mid f \text{ is a function}\}$ with pointwise addition and pointwise multiplication. Prove that $\mathcal{F}(S)$ is a ring.
- (2) Let A be a ring. Prove that $I = \{p(x) \in A[x] \mid p(0) = 0\}$ is an ideal of $A[x]$.
- (3) If A and B are rings and $f : A \rightarrow B$ is a ring homomorphism, then prove that $\ker f$ is an ideal of A .
- (4) Let A be a ring and \mathfrak{p} be a prime ideal of A . Prove that $\mathfrak{P} = \left\{ \sum_{i=0}^n a_i x^i \in A[x] \mid a_i \in \mathfrak{p} \right\}$ is a prime ideal of $A[x]$.
- (5) Prove that \mathbb{Z}_9 is a local ring. Write down the maximal ideal of \mathbb{Z}_9 .