NPTEL COURSE - Introduction to Commutative Algebra

Assignment - Week 11

(1) Let A be a ring and M be an A-module. Prove that

$$\mathrm{Ass}(M) = \{\mathfrak{p} \in \mathrm{Spec}(A) \mid \mathrm{Hom}_A(A/\mathfrak{p}, M)_{\mathfrak{p}} \neq 0\}.$$

(2) For an ideal $I \subset A$, prove that if $I = \mathrm{rad}(I)$, then I has no embedded primary components.

(3) Let $(A, \mathfrak{m})$ be a Noetherian local ring and I be an ideal in A. Prove that $\cap_{n=1}^{\infty} I^n = 0$.

(4) Prove that a finitely generated $\mathbb{Z}$-module M is Artinian if and only if M is finite.