

NPTEL COURSE - Introduction to Commutative Algebra

Assignment - Week 10

- (1) Let  $A$  be a Noetherian ring,  $B$  a finitely generated  $A$ -algebra,  $G$  a finite group of  $A$ -automorphisms of  $B$  and  $B^G := \{x \in B \mid f(x) = x \text{ for all } f \in G\}$ . Show that  $B^G$  is a finitely generated  $A$ -algebra.
- (2) If  $n\mathbb{Z} \subset \mathbb{Z}$  is an irreducible ideal, then prove that  $n = p^r\mathbb{Z}$  for some prime  $p$  and a positive integer  $r$ .
- (3) Find a minimal primary decomposition of  $(x^3, x^2y^2, xz^3) \subset k[x, y, z]$ . List the isolated and embedded prime ideals.