Assignment 6

1. Let x, y be solutions to the system of equations $2x + 5y = 1$ and $x + 2y = 3$. Then the value of $x + y$ is:

2. Let $A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$. Let $f(x), g(x)$ be nonzero polynomials such that $\deg f > \deg g$ and $f(A) = g(A)$. Choose all the true statements from the list below:

   - $f(A)$ is a diagonal matrix.
   - $\deg f \geq 2$.
   - $\deg g \geq 2$.
   - $\det(f(A)) = f(6)$.

3. If A is a 2×2 matrix such that $A^2 = \begin{pmatrix} 4 & 0 \\ 0 & 9 \end{pmatrix}$, then $A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ or $A = \begin{pmatrix} -2 & 0 \\ 0 & -3 \end{pmatrix}$.

   - True.
   - False.

4. If A, B are 2×2 matrices such that $AB = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$, then both A and B must be diagonal matrices.

   - True.
   - False.

5. If A, B are 2×2 matrices such that $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, then at least one of A or B must be $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

   - True.
   - False.

6. If A, B are 2×2 matrices such that $AB = \begin{pmatrix} 0 & 5 \\ 0 & 1 \end{pmatrix}$, then $\det A$ or $\det B$ equals zero.

   - True.
   - False.

7. Let $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. What is the smallest value of d for which there is a polynomial $f(x)$ of degree d such that $f(A) = 0$?