Assignment 5

For problems 1-5, state True or False.

1. If $f(x,y) = (-y,x)$ and $g(x,y) = (\frac{x-y}{\sqrt{2}}, \frac{x+y}{\sqrt{2}})$, then the composition of g with itself produces f, i.e., $g \circ g = f$.

2. Let f be an arbitrary affine transformation of $\mathbb{R}^2$. Let $f(1,1) = (p,q)$. Then $f(2,2) = (2p, 2q)$.

3. Let f be an arbitrary linear transformation of $\mathbb{R}^2$. Let $f(1,1) = (p,q)$. Then $f(2,2) = (2p, 2q)$.

4. Let f be an arbitrary linear transformation of $\mathbb{R}^2$. The image of the unit circle $x^2 + y^2 = 1$ under f is a circle.

5. There is a unique linear transformation of $\mathbb{R}^2$ which maps the X-axis to the line $y = 2x$, and the Y-axis to the line $y = x$.

6. Let S be the square with vertices $(0,0), (1,0), (0,1), (1,1)$. The number of linear transformations of $\mathbb{R}^2$ which map S to itself is:

   - 1
   - 2
   - 3
   - infinitely many.

7. Let $a > 0$ and define the linear transformation $f(x,y) = (ax - y, ax + y)$. If f dilates areas of regions of $\mathbb{R}^2$ by a factor of 6, then the value of a is: