Assignment 4

1. If $f(x)$ is a polynomial of degree 2 such that (i) all coefficients in f are ≥ 0, (ii) $f(n) \in \mathbb{Z}$ for all $n \in \mathbb{Z}$ and (iii) $f(0) = f(1),\ f(\frac{1}{2}) = 0$. Then, the smallest possible value of $f(2)$ is:

2. If $f(x)$ is a polynomial of degree ≤ 2 such that $f(x) \in \mathbb{Z}$ for $x = 0, 1, 2$. Then $f(n) \in \mathbb{Z}$ for all $n \in \mathbb{Z}$.

   - True.
   - False.

3. If $f(x)$ is a polynomial of degree ≤ 2 such that $f(x) \in \mathbb{Z}$ for $x = 0, 2, 4$. Then $f(n) \in \mathbb{Z}$ for all $n \in \mathbb{Z}$.

   - True.
   - False.

4. If $f(n) \in \mathbb{Z}$ for all $n \in \mathbb{Z}$, then we will call $f(x)$ an integral polynomial. Suppose f, g are integral polynomials, then which of the folllowing are integral polynomials ?

   - $f(x) + g(x)$.
   - $f(x)g(x)$.
   - $f(g(x))$.
   - $\frac{f(x)(g(x)-1)}{2}$.

5. If $f(x) + g(x)$ is an integral polynomial, then so are $f(x)$ and $g(x)$.

   - True.
   - False.

6. If $f(x)g(x)$ is an integral polynomial, then so are $f(x)$ and $g(x)$.

   - True.
   - False.

7. If $f(g(x))$ is an integral polynomial, then so are $f(x)$ and $g(x)$.

   - True.
   - False.