Assignment 3: Solutions

1. The number of permutations of $1, 2, \cdots, 6$ with cycle type $3+3$ is (please enter only the final numerical answer): $\boxed{40}$

   Three letters for the first 3-cycle can be chosen in ${}^6C_3$ ways. Once we fix them, the three letters in the second 3-cycle get fixed. Now the first set of three letters gives rise to two 3-cycles, similarly the second set of three letters gives two 3-cycles. Therefore the number of permutations of 3+3 type looks like ${}^6C_3 \cdot 2 \cdot 2$. But we have counted $(abc)(def)$ and $(def)(abc)$ differently. Hence the answer is $\dfrac{{}^6C_3 \cdot 2 \cdot 2}{2} = 40$

2. The number of permutations of $1, 2, \cdots, 6$ with cycle type $3+2+1$ is: $\boxed{120}$

   First choose 3 numbers for the 3-cycle; this can be done in ${}^6C_3 = 20$ ways. Arranging these 3 numbers into a cycle can be done in 2! ways. Next, from the remaining 3 numbers, choose 2 of them; this can be done in ${}^3C_2 = 3$ ways; arranging them in a cycle can only be done in $1! = 1$ way. There is no choice left for the 1-cycle. Thus, total number equals $20 \times 2 \times 3 = 120$ ways.

3. Let σ be the following permutation of $1, 2, \cdots, 5$ (written in cycle notation): $\sigma = (1\ 5)(2\ 4)(3)$. The number of inversions (crossings) of σ is: $\boxed{10}$

4. Let π be the following permutation of $1, 2, \cdots, 5$ (written in cycle notation): $\pi = (1\ 3\ 5)(2\ 4)$. The number of inversions of π is: $\boxed{7}$

5. Let n be a natural number, and let S_n denote the set of all permutations of $1, 2, \cdots, n$. What is the maximum possible number of crossings a permutation in S_n could have ?

   - $\dfrac{n(n+1)}{2}$.
   - $\dfrac{n(n-1)}{2}$.
   - n^2.
   - n.

   $n, (n-1), ..., 2, 1$ (the numbers written in descending order) is the unique permutation in which every pair $i < j$ is an inversion.

6. Referring back to the previous question, how many different permutations in S_n have this maximum number of crossings ?

   - 1.
   - 2.
   - $n!$
   - n.

Refer the previous problem.

7. Let σ, π be arbitrary permutations in S_n (where S_n is as above). Then $\sigma \circ \pi$ and $\pi \circ \sigma$ have the same number of crossings.

   - True.
   - False.

Find some easy examples in S_3.