Solutions - Assignment 2

1. The number of words of length 10 in the letters a, b, c in which a occurs 5 times, b occurs 3 times and c occurs 2 times is:

   a) ${}^{10}C_5\ {}^{5}C_3$

   b) ${}^{10}C_8$

   c) ${}^{8}C_3\ {}^{5}C_2$

   d) 10!

   The positions of the letter a to form a word of length 10 can be chosen in ${}^{10}C_5$ ways, The positions of the letter b from the remaining 5 positions can be chosen in ${}^{5}C_3$ ways. Now the remaining 2 positions are fixed for the letter c and hence there is no choice for c.

2. The total number of sequences of length 10 containing zeros and ones is:

   a) ${}^{10}C_2$

   b) 10!

   c) 2^{10}

   d) None of the above

3. A bag contains a large number of balls of each of 4 colours (Red, Blue, Green, Yellow). The number of ways of choosing 10 balls from this bag such that at least one red ball is chosen is

   a) ${}^{13}C_3$

   b) ${}^{12}C_3$

   c) 4^{10}

   d) 4^9

   Since we are forced to take a red ball, the problem is equivalent to the number of ways of choosing 9 balls from the bag.

4. Let a =number of monomials of degree 4 in 7 variables and $b =$ number of monomials of degree 6 in 5 variables. Then

   a) $a < b$

   b) $a > b$

   c) $a = b$

   d) None of the above

We have $a = b = {}^{10}C_4 = {}^{10}C_6$.

5. A sequence which only consists of zeros and ones is called a 0-1 sequence. Recall also that the Fibonacci numbers $F_n, n \geq 1$ are defined by $F_1 = 1,\ F_2 = 1,\ F_n = F_{n-1} + F_{n-2}$ where $n \geq 3$. The number of 0-1 sequences of length n which do not contain two successive zeros is:

   a) F_{n+2}

   b) F_{n-1}

   c) F_n

   d) none of the above

   By symmetry (interchange 0 and 1), the number of sequences of zeros and ones which have lengh n and do not contain two successive zeros is equal to the number of sequences of zeros and ones which have lengh n and do not contain two successive ones.

6. The number of $0-1$ sequences of length n which contain at least one pair of successive ones is

   a) ${}^{n}C_2$

   b) $2^n - F_{n+2}$

   c) F_n

   d) none of the above

   (The number of $0-1$ sequences of length n which do not contain two successive ones) + (The number of $0-1$ sequences of length n which contain at least one pair of successive ones) $= 2^n$.

7. The number of $0-1$ sequences of length n in which a 1 never occurs after a 0 is

   a) $n+1$

   b) F_{n+2}

   c) $2^n - F_{n+2}$

   d) none of the above

   Consider a $0-1$ sequence of length n in which a 1 never occurs after a 0. If there is a 1 in such a sequence, then all the preceeding entries also have to be ones. Hence the choices look like : (for $n = 4$) $(0, 0, 0, 0)$, $(1, 0, 0, 0)$,$(1, 1, 0, 0)$,$(1, 1, 1, 0)$,$(1, 1, 1, 1)$. In general, the number of such sequences is $n + 1$.

8. The number of $0-1$ sequences containing 3 zeros and 7 ones in which no two zeros occur successively is

   a) F_{12}
   b) ${}^{7}C_{3}$
   c) ${}^{8}C_{3}$
   d) none of the above

   Since no two zeros can occur successively, there will be at least one 1 between any two zeros. So, the zeros can occupy three of the eight "gaps" (counting the gaps at the very beginning and at the very end) in a string of seven consecutive ones.