Assignment 1

1. Let f, g be polynomials with $\deg f = 3, \deg g \leq 3$. Suppose $f(x) = g(x)$ for $x = 1, 2, 3, 4$. What is the degree of g ?

   - 0
   - 1
   - 2
   - 3
   - cannot say.

2. Let f be a polynomial such that $f(1) = 2, f(2) = 7, f(3) = 9$. Find the remainder obtained on division of $f(x)$ by $x - 2$.

3. Let f be a polynomial such that $f(n) = 2^n$ for $n = 0, 1, 2, 3$. What is the minimum possible value of the degree of f ?

4. Let f be a polynomial such that $f(n) = \frac{n(n-1)}{2}$ for $n = 0, 1, 2, 3$. What is the minimum possible value of the degree of f ?

5. Let f, g, h be polynomials of degree ≤ 1. Let $k(x) = f(x)g(x^2)h(x^4)$. If $\deg k = 5$, what is the value of $\deg f + 3 \deg g + 9 \deg h$?

6. Let f be a polynomial of degree ≤ 2 satisfying $f(1) = 1, f'(1) = 2, f''(1) = 3$. What is the value of $f(1)$?

7. Let $A = (1, 1)$ and $B = (1, 1)$ be points in $\mathbb{R}^2$. Let D be the point in $\mathbb{R}^2$ such that the vector $\vec{OD}$ satisfies $\vec{OD} \cdot \vec{OA} = 3$ and $\vec{OD} \cdot \vec{OB} = -1$. What is the value of $\vec{OD} \cdot \vec{OD}$?

8. Let f, g be polynomials of degrees a, b respectively. Let $h(x) = f(g(x))$. The degree of h is:

   - ab
   - $a + b$
   - a^b
   - b^a

9. Let f, g, h be nonzero polynomials such that $f(x) - g(x) = h(x)$ and $\deg f = \deg h$. Pick the true statement:

   - $\deg g \leq \deg f$.
   - $\deg g > \deg f$.
   - $\deg g$ has no relation to $\deg f$.

10. Let f, g, h be polynomials such that $f(x) = g(x) + x^3\, h(x)$. Then $f^{(j)}(0) = g^{(j)}(0)$ for

- $j = 0$.
- $j = 1$.
- $j = 2$.
- all the above.