# Unit 11 - Week 9



## Assignment 9

**The due date for submitting this assignment has passed.**
**As per our records you have not submitted this assignment.**

**Due on 2020-11-18, 23:59 IST.**

1) Let $\mathbf{x} = (x_1, x_2)^T,\ \mathbf{y} = (y_1, y_2)^T \in \mathbb{R}^2$. Then, $\langle \mathbf{x}, \mathbf{y} \rangle = 3x_1y_1 - x_1y_2 - x_2y_1 + x_2y_2$ defines an inner product. If, under this inner product, $\theta$ is the angle between $\mathbf{e}_1 = (1,0)^T$ and $\mathbf{e}_2 = (0,1)^T$ then the value of $\frac{\sqrt{3}}{2}|\cos\theta|$ equals ....

No, the answer is incorrect.
Score: 0
Accepted Answers:
*(Type: Range) .49,.51*

*1 point*

2) Consider $\mathbb{R}^5$ with the standard inner product. Let $\mathbf{u} = (-1, 1, 2, 3, 7)^T \in \mathbb{R}^5$. Then, the value of $|2\alpha|$ such that $\|\alpha\mathbf{u}\| = 1$ equals....

No, the answer is incorrect.
Score: 0
Accepted Answers:
*(Type: Range) .24,.26*

*1 point*

3) Consider the two statements given below: *1 point*
(I) $\|\mathbf{x}+\mathbf{y}\|^2 + \|\mathbf{x}-\mathbf{y}\|^2 = 2(\|\mathbf{x}\|^2 + \|\mathbf{y}\|^2)$, for all $\mathbf{x}^T, \mathbf{y}^T \in \mathbb{R}^n$.
(II) $\left|\|\mathbf{x}\| - \|\mathbf{y}\|\right| \le \|\mathbf{x}-\mathbf{y}\|$, for all $\mathbf{x}^T, \mathbf{y}^T \in \mathbb{R}^n$.
Then

- Statement I is TRUE but Statement II is FALSE
- Statement I is FALSE but Statement II is TRUE
- both Statement I and Statement II are TRUE
- neither Statement I is TRUE nor Statement II is TRUE

No, the answer is incorrect.
Score: 0
Accepted Answers:
*both Statement I and Statement II are TRUE*

4) Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ be a matrix such that $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{y}^T A\mathbf{x}$ defines an inner product in $\mathbb{R}^2$. Suppose, under the given inner product $\|\mathbf{u}\| = 1, \|\mathbf{v}\| = 1$ and $\langle \mathbf{u}, \mathbf{v} \rangle = 0$, for $\mathbf{u} = (1,2)^T$ and $\mathbf{v} = (2,-1)^T \in \mathbb{R}^2$.
Then, the value of
$5(a+c)+b$ equals ...

No, the answer is incorrect.
Score: 0
Accepted Answers:
*(Type: Range) 1.9,2.1*

*1 point*

5) Let the length of the sides of a triangle be $a, b, c \in \mathbb{R}$ and that of the median be $d \in \mathbb{R}$. If the median is drawn on the side with length $a$ then *1 point*

- $b^2 + c^2 = d^2 + a^2$
- $b^2 + c^2 = 2\left(d^2 + \left(\frac{a}{2}\right)^2\right)$
- $b^2 + c^2 = 2\left(a^2 + \left(\frac{d}{2}\right)^2\right)$
- $b^2 + c^2 = 2d^2 + a^2$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$b^2 + c^2 = 2\left(d^2 + \left(\frac{a}{2}\right)^2\right)$

6) Under the standard inner product in $\mathbb{M}_{m,n}(\mathbb{R})$, $\mathbb{R}^m$ and $\mathbb{R}^n$, consider the two statements *1 point*
(I) for $A \in \mathbb{M}_{m,n}(\mathbb{R})\$, \$\|A\|^2 = \text{trace}(A^TA) = \sum_{k=1}^{m} \|A[k,:]\|^2 = \sum_{\ell=1}^{n} \|A[:,\ell]\|^2$.
(II) for $A \in \mathbb{M}_{m,n}(\mathbb{R})$ and $\mathbf{x} \in \mathbb{R}^n$, $\|A\mathbf{x}\| \le \|A\| \cdot \|\mathbf{x}\|$.

- Statement I is TRUE but Statement II is FALSE
- Statement I is FALSE but Statement II is TRUE
- both Statement I and Statement II are TRUE
- neither Statement I is TRUE nor Statement II is TRUE

No, the answer is incorrect.
Score: 0
Accepted Answers:
*both Statement I and Statement II are TRUE*

7) Which among the following is an INCORRECT Option? *1 point*

- $a^2 + b^2 \ge 2|ab|$ for all $a, b \in \mathbb{R}$
- There exists a choice of $a, b \in \mathbb{R}$ such that $(a^2+b^2)\left(\frac{1}{a^2} + \frac{1}{b^2}\right) < 4$
- $(a^2+b^2)\left(\frac{1}{a^2} + \frac{1}{b^2}\right) \ge 4$ for all $a, b \in \mathbb{R}$
- $e^a + e^b \ge 2\sqrt{e^{a+b}}$ for all $a, b \in \mathbb{R}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
*There exists a choice of $a, b \in \mathbb{R}$ such that $(a^2+b^2)\left(\frac{1}{a^2} + \frac{1}{b^2}\right) < 4$*

8) Let $S = \{\mathbf{e}_1 + \mathbf{e}_4, -\mathbf{e}_1 + 3\mathbf{e}_2 - \mathbf{e}_3\} \subset \mathbb{R}^4$. Then *1 point*

- $S^\perp = \text{LS}\left([3,1,0,-3]^T, [0,-1,3,0]^T\right)$
- $S^\perp = \text{LS}\left([1,0,0,-1]^T, [0,1,3,0]^T\right)$
- $S^\perp = \text{LS}\left([-3,1,0,3]^T, [0,-1,3,0]^T\right)$
- $S^\perp = \text{LS}\left([3,1,0,-3]^T, [0,1,3,0]^T\right)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$S^\perp = \text{LS}\left([3,1,0,-3]^T, [0,1,3,0]^T\right)$

9) Let $\mathbf{u} = (1,1,1,1)^T, \mathbf{v} = (1,1,-1,0)^T$, $\mathbf{w} = (1,1,0,-1)^T$ and $\mathbf{z} = (3,3,-5,-1)^T \in \mathbb{R}^4$. Then, which among the following is an INCORRECT Option? *0 points*

- $\frac{1}{4}\mathbf{z}$ is the component of $\mathbf{v}$ which is orthogonal to $\mathbf{u}$
- $\frac{7}{44}\mathbf{z}$ is the component of $\mathbf{w}$ which is parallel to $\mathbf{z}$
- $\frac{5}{11}(1,1,2,-4)^T$ is the component of $\mathbf{v}$ which is orthogonal to both $\mathbf{u}$ and $\mathbf{z}$
- $\frac{3}{11}(1,1,2,-4)^T$ is the component of $\mathbf{v}$ which is orthogonal to both $\mathbf{u}$ and $\mathbf{z}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
*$\frac{5}{11}(1,1,2,-4)^T$ is the component of $\mathbf{v}$ which is orthogonal to both $\mathbf{u}$ and $\mathbf{z}$*

10) Consider $\mathbb{R}^n$ with the standard inner product. If $\mathbf{u} \in \mathbb{R}^n$ then, which among the following is an INCORRECT Option? *0 points*

- If $\mathbf{u}^\perp = \{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{u}^T\mathbf{v} = 0\}$ then $\dim(\mathbf{u}^\perp) = n-1$
- Let $0 \ne \beta \in \mathbb{R}$. Then $W = \{\mathbf{v} \in \mathbb{R}^n : \mathbf{u}^T\mathbf{v} = \beta\}$ is NOT a subspace of $\mathbb{R}^n$
- $\mathbb{R}^n = LS(\mathbf{u}, \mathbf{u}^\perp)$
- Let $\mathbf{v} \in \mathbb{R}^n$. Then $\mathbf{v} = \mathbf{v}_0 + \mathbf{u}$ for a vector $\mathbf{v}_0 \in \mathbf{u}^\perp$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
*Let $\mathbf{v} \in \mathbb{R}^n$. Then $\mathbf{v} = \mathbf{v}_0 + \mathbf{u}$ for a vector $\mathbf{v}_0 \in \mathbf{u}^\perp$.*