Unit 10 - Week 8

Assignment 8

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-11-11, 23:59 IST.

1) Let $\mathcal{B} = \left(\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}\right)$ and $\mathcal{C} = \left(\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}\right)$ be two ordered bases of $\mathbb{R}^3$. If $\mathbf{v} = \begin{bmatrix}1\\2\\3\end{bmatrix}$ then *1 point*

- $[\mathbf{v}]_\mathcal{B} = [3 \ \ -1 \ \ -1]^T$ and $[\mathbf{v}]_\mathcal{C} = [0 \ 1 \ 2]^T$
- $[\mathbf{v}]_\mathcal{B} = [3 \ \ -1 \ \ 1]^T$ and $[\mathbf{v}]_\mathcal{C} = [1 \ 0 \ 2]^T$
- $[\mathbf{v}]_\mathcal{B} = [3 \ \ -1 \ \ -1]^T$ and $[\mathbf{v}]_\mathcal{C} = [1 \ 0 \ 2]^T$
- $[\mathbf{v}]_\mathcal{B} = [3 \ \ -1 \ \ 1]^T$ and $[\mathbf{v}]_\mathcal{C} = [0 \ 1 \ 2]^T$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$[\mathbf{v}]_\mathcal{B} = [3 \ \ -1 \ \ -1]^T$ *and* $[\mathbf{v}]_\mathcal{C} = [1 \ 0 \ 2]^T$

2) Let $\mathbb{V} = \{(x, y, z, w) \in \mathbb{R}^4 : x = y\}$ be a subspace of $\mathbb{R}^4$. If $\mathcal{B} = \left(\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\\0\end{bmatrix}\right)$ is an ordered basis of $\mathbb{V}$ then for $\mathbf{v} = \begin{bmatrix}1\\1\\2\\3\end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix}1\\1\\-2\\-3\end{bmatrix}$ *1 point*

- $[\mathbf{v}]_\mathcal{B} = [3 \ \ -1 \ \ 1]^T$ and $[\mathbf{w}]_\mathcal{B} = [-3 \ \ -1 \ \ 2]^T$
- $[\mathbf{v}]_\mathcal{B} = [3 \ \ -1 \ \ -1]^T$ and $[\mathbf{w}]_\mathcal{B} = [-3 \ 1 \ 3]^T$
- $[\mathbf{v}]_\mathcal{B} = [3 \ \ -1 \ \ 1]^T$ and $[\mathbf{w}]_\mathcal{B} = [3 \ 1 \ 2]^T$
- $[\mathbf{v}]_\mathcal{B} = [3 \ \ -1 \ \ -1]^T$ and $[\mathbf{w}]_\mathcal{B} = [-3 \ \ -1 \ \ 3]^T$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$[\mathbf{v}]_\mathcal{B} = [3 \ \ -1 \ \ -1]^T$ *and* $[\mathbf{w}]_\mathcal{B} = [-3 \ 1 \ 3]^T$

3) Let $\mathcal{B} = \left(\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}\right)$ and $\mathcal{C} = \left(\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}\right)$ be two ordered bases of $\mathbb{R}^3$. If $[\mathbf{v}]_\mathcal{B} = \begin{bmatrix}1\\2\\3\end{bmatrix}$ and $[\mathbf{w}]_\mathcal{C} = \begin{bmatrix}1\\2\\3\end{bmatrix}$ then *1 point*

- $\mathbf{v} = [6 \ 3 \ 1]^T$ and $\mathbf{w} = [3 \ 5 \ 4]^T$
- $\mathbf{v} = [6 \ 3 \ 2]^T$ and $\mathbf{w} = [3 \ 5 \ 5]^T$
- $\mathbf{v} = [3 \ 5 \ 4]^T$ and $\mathbf{w} = [6 \ 3 \ 1]^T$
- $\mathbf{v} = [3 \ 5 \ 5]^T$ and $\mathbf{w} = [6 \ 3 \ 2]^T$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mathbf{v} = [6 \ 3 \ 1]^T$ *and* $\mathbf{w} = [3 \ 5 \ 4]^T$

4) Recall that every $A \in \mathbb{M}_3(\mathbb{R})$ can be written as a sum of Hermitian and skew-Hermitian matrices. Thus $\mathbb{M}_3(\mathbb{R}) = \mathbb{U} + \mathbb{W}$, where $\mathbb{U} = \{A \in \mathbb{M}_3(\mathbb{R}) | A^T = A\}$ and $\mathbb{W} = \{A \in \mathbb{M}_3(\mathbb{R}) | A^T = -A\}$. Let $A = \begin{bmatrix}1 & 2 & 3\\2 & 1 & 3\\3 & 1 & 4\end{bmatrix}$. Then $A = X + Y$ for some $X \in \mathbb{U}$ and $Y \in \mathbb{W}$. If $\mathcal{C} = (\mathbf{e}_{11}, \mathbf{e}_{12} + \mathbf{e}_{21}, \mathbf{e}_{13} + \mathbf{e}_{31}, \mathbf{e}_{22}, \mathbf{e}_{23} + \mathbf{e}_{32}, \mathbf{e}_{33})$ and $\mathcal{D} = (\mathbf{e}_{12} - \mathbf{e}_{21}, \mathbf{e}_{13} - \mathbf{e}_{31}, \mathbf{e}_{23} - \mathbf{e}_{32})$ are, respectively, the bases of $\mathbb{U}$ and $\mathbb{W}$ then *1 point*

- $[X]_\mathcal{C} = [1 \ \ 2 \ \ 3 \ \ 1 \ \ 2 \ \ 4]^T$ and $[Y]_\mathcal{D} = [0 \ \ 0 \ \ 1]^T$
- $[X]_\mathcal{C} = [1 \ \ 2 \ \ 3 \ \ 1 \ \ 2 \ \ 3]^T$ and $[Y]_\mathcal{D} = [0 \ \ 0 \ \ -1]^T$
- $[X]_\mathcal{C} = [1 \ \ 2 \ \ 3 \ \ 1 \ \ 2 \ \ 3]^T$ and $[Y]_\mathcal{D} = [0 \ \ 0 \ \ 1]^T$
- $[X]_\mathcal{C} = [1 \ \ 2 \ \ 3 \ \ 1 \ \ 2 \ \ 4]^T$ and $[Y]_\mathcal{D} = [0 \ \ 0 \ \ -1]^T$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$[X]_\mathcal{C} = [1 \ \ 2 \ \ 3 \ \ 1 \ \ 2 \ \ 4]^T$ *and* $[Y]_\mathcal{D} = [0 \ \ 0 \ \ 1]^T$

5) Let $\mathcal{A} = \left(\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}\right)$ and $\mathcal{B} = \left(\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}\right)$ be two ordered bases of $\mathbb{R}^3$. Then *1 point*

- $[\mathcal{A}]_\mathcal{B} = \begin{bmatrix}1/2 & 0 & 1\\1/2 & 0 & 0\\1 & 1 & 0\end{bmatrix}$ and $[\mathcal{B}]_\mathcal{A} = \begin{bmatrix}0 & 2 & 0\\0 & -2 & 1\\1 & 1 & 0\end{bmatrix}$
- $[\mathcal{A}]_\mathcal{B} = \begin{bmatrix}1/2 & 0 & 1\\1/2 & 0 & 0\\1 & 1 & 0\end{bmatrix}$ and $[\mathcal{B}]_\mathcal{A} = \begin{bmatrix}0 & 2 & 0\\0 & 2 & 1\\1 & 1 & 0\end{bmatrix}$
- $[\mathcal{A}]_\mathcal{B} = \begin{bmatrix}-1/2 & 0 & 1\\1/2 & 0 & 0\\1 & 1 & 0\end{bmatrix}$ and $[\mathcal{B}]_\mathcal{A} = \begin{bmatrix}0 & 2 & 0\\0 & -2 & 1\\1 & 1 & 0\end{bmatrix}$
- $[\mathcal{A}]_\mathcal{B} = \begin{bmatrix}-1/2 & 0 & 1\\1/2 & 0 & 0\\1 & 1 & 0\end{bmatrix}$ and $[\mathcal{B}]_\mathcal{A} = \begin{bmatrix}0 & 2 & 0\\0 & 2 & 1\\1 & 1 & 0\end{bmatrix}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$[\mathcal{A}]_\mathcal{B} = \begin{bmatrix}-1/2 & 0 & 1\\1/2 & 0 & 0\\1 & 1 & 0\end{bmatrix}$ *and* $[\mathcal{B}]_\mathcal{A} = \begin{bmatrix}0 & 2 & 0\\0 & -2 & 1\\1 & 1 & 0\end{bmatrix}$

6) Let $T \in \mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$ be defined by $T((x, y, z)^T) = (x + y - z, x + z)^T$. Then, *1 point*

- $[T] = [[T(\mathbf{e}_1)], [T(\mathbf{e}_2)], [T(\mathbf{e}_3)]] = \left[\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}-1\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}\right] = \begin{bmatrix}1 & -1 & 1\\1 & 1 & 0\end{bmatrix}$
- $[T] = [[T(\mathbf{e}_1)], [T(\mathbf{e}_2)], [T(\mathbf{e}_3)]] = \left[\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}-1\\1\end{bmatrix}\right] = \begin{bmatrix}1 & 1 & -1\\1 & 0 & 1\end{bmatrix}$
- $[T] = [[T(\mathbf{e}_1)], [T(\mathbf{e}_2)], [T(\mathbf{e}_3)]] = \left[\begin{bmatrix}-1\\1\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}\right] = \begin{bmatrix}-1 & 1 & 1\\1 & 1 & 0\end{bmatrix}$
- $[T] = [[T(\mathbf{e}_1)], [T(\mathbf{e}_2)], [T(\mathbf{e}_3)]] = \left[\begin{bmatrix}-1\\1\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}\right] = \begin{bmatrix}-1 & 1 & 1\\1 & 0 & 1\end{bmatrix}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$[T] = [[T(\mathbf{e}_1)], [T(\mathbf{e}_2)], [T(\mathbf{e}_3)]] = \left[\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}-1\\1\end{bmatrix}\right] = \begin{bmatrix}1 & 1 & -1\\1 & 0 & 1\end{bmatrix}$

7) Define $T \in \mathcal{L}(\mathbb{C}^3)$ by $T(\mathbf{x}) = \mathbf{x}$, for all $\mathbf{x} \in \mathbb{C}^3$. If $\mathcal{A} = (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ and $\mathcal{B} = ((1,0,0), (1,1,0), (1,1,1))$ are the ordered bases of $\mathbb{R}^3$ then *1 point*

- $T[\mathcal{A}, \mathcal{B}] = \begin{bmatrix}1 & -1 & 0\\0 & 1 & -1\\0 & 0 & 1\end{bmatrix}$ and $T[\mathcal{B}, \mathcal{A}] = \begin{bmatrix}1 & 1 & 1\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$
- $T[\mathcal{B}, \mathcal{A}] = \begin{bmatrix}1 & -1 & 0\\0 & 1 & -1\\0 & 0 & 1\end{bmatrix}$ and $T[\mathcal{A}, \mathcal{B}] = \begin{bmatrix}1 & 1 & 1\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$
- $T[\mathcal{A}, \mathcal{B}] = \begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$ and $T[\mathcal{B}, \mathcal{A}] = \begin{bmatrix}1 & -1 & 1\\0 & 1 & -1\\0 & 0 & 1\end{bmatrix}$
- $T[\mathcal{B}, \mathcal{A}] = \begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$ and $T[\mathcal{A}, \mathcal{B}] = \begin{bmatrix}1 & -1 & 1\\0 & 1 & -1\\0 & 0 & 1\end{bmatrix}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$T[\mathcal{A}, \mathcal{B}] = \begin{bmatrix}1 & -1 & 0\\0 & 1 & -1\\0 & 0 & 1\end{bmatrix}$ *and* $T[\mathcal{B}, \mathcal{A}] = \begin{bmatrix}1 & 1 & 1\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$

8) Let $T \in \mathcal{L}(\mathbb{R}^2)$ represent the reflection about the line $y = mx$. Then *1 point*

- $[T] = \dfrac{1}{1+m^2}\begin{bmatrix}m^2-1 & 2m\\2m & m^2-1\end{bmatrix}$
- $[T] = \dfrac{1}{1+m^2}\begin{bmatrix}1-m^2 & 2m\\2m & m^2-1\end{bmatrix}$
- $[T] = \dfrac{1}{1+m^2}\begin{bmatrix}2m & m^2-1\\m^2-1 & -2m\end{bmatrix}$
- $[T] = \dfrac{1}{1+m^2}\begin{bmatrix}-2m & 1-m^2\\1-m^2 & 2m\end{bmatrix}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$[T] = \dfrac{1}{1+m^2}\begin{bmatrix}1-m^2 & 2m\\2m & m^2-1\end{bmatrix}$

9) Let $\mathbb{V}$ be a vector space with $\dim(\mathbb{V}) = n$. Let $T \in \mathcal{L}(\mathbb{V})$ satisfy $T^{n-1} \neq \mathbf{0}$ but $T^n = \mathbf{0}$. Then, which among the following is INCORRECT Option? *1 point*

- there exists $\mathbf{u} \in \mathbb{V}$ such that $T^{n-1}(\mathbf{u}) \neq \mathbf{0}$ but $T^n(\mathbf{u}) = \mathbf{0}$
- $\{\mathbf{u}, T(\mathbf{u}), \ldots, T^{n-1}(\mathbf{u})\}$ is a linearly dependent set in $\mathbb{V}$
- there exists a basis $\mathcal{B}$ of $\mathbb{V}$ such that $T[\mathcal{B}, \mathcal{B}] = \begin{bmatrix}0 & 0 & 0 & \cdots & 0\\1 & 0 & 0 & \cdots & 0\\0 & 1 & 0 & \cdots & 0\\\vdots & & \ddots & \ddots & \vdots\\0 & 0 & \cdots & 1 & 0\end{bmatrix}$
- there exists a basis $\mathcal{B}$ of $\mathbb{V}$ such that $T[\mathcal{B}, \mathcal{B}] = \begin{bmatrix}0 & 1 & 0 & \cdots & 0 & 0\\0 & 0 & 1 & \cdots & 0 & 0\\0 & 0 & 0 & \ddots & 0 & 0\\\vdots & & \ddots & \ddots & \ddots & \vdots\\0 & 0 & \cdots & \ddots & 0 & 1\\0 & 0 & \cdots & & 0 & 0\end{bmatrix}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\{\mathbf{u}, T(\mathbf{u}), \ldots, T^{n-1}(\mathbf{u})\}$ *is a linearly dependent set in* $\mathbb{V}$