Unit 9 - Week 7

Assignment 7

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-11-04, 23:59 IST.

For $f : \mathbb{A} \to \mathbb{B}$, $\text{Rng}(f) = \text{Im}(f) = \{f(\mathbf{a}) : \mathbf{a} \in \mathbb{A}\}$$.

1) Consider the functions $f_1, f_2, f_3 : \mathbb{R}^2 \to \mathbb{R}^3$ defined by *1 point*
$f_1((x,y)^T) = (x+y+1, 2x-y, x+3y)^T$, $f_2((x,y)^T) = (x+y, x-y, x^2-y^2)^T$
and $f_3((x,y)^T) = (x+y, x-y, |x|)^T$. Then

- f_1 and f_2 are linear transformations.
- f_2 and f_3 are linear transformations
- neither f_2 nor f_3 are linear transformations
- f_1 is a linear transformation

No, the answer is incorrect.
Score: 0
Accepted Answers:
neither f_2 nor f_3 are linear transformations

2) Let $T : \mathbb{R}^n \to \mathbb{R}$ be a linear transformation. Then *1 point*

- there exists a Non-zero vector $\mathbf{a} \in \mathbb{R}^n$ such that $T(\mathbf{x}) = \mathbf{a}^T\mathbf{x}$
- there exists a unique vector $\mathbf{a} \in \mathbb{R}^n$ such that $T(\mathbf{x}) = \mathbf{a}^T\mathbf{x}$
- the dimension of $\text{Null}(T) \leq n-2$
- the dimension of $\text{Im}(T) = 1$, where $\text{Im}(T)$ denotes the range space of T

No, the answer is incorrect.
Score: 0
Accepted Answers:
there exists a unique vector $\mathbf{a} \in \mathbb{R}^n$ such that $T(\mathbf{x}) = \mathbf{a}^T\mathbf{x}$

3) Let $f_1, f_2 : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by *1 point*
$f_1((x,y,z)^T) = (2x+3y+4z, x+y+z, x+y+3z)^T$
and
$f_2((x,y,z)^T) = (2x+y+3z, 4x-y+3z, 3x-2y+5z)^T$.
Then

- there is a unique value of k for which $(9,3,k)^T \in \text{Im}(f_1)$
- there is a unique value of k for which $(9,3,k)^T \in \text{Im}(f_2)$
- there exist Non-zero vectors $\mathbf{x}$ and $\mathbf{y}$ such that $f_2(\mathbf{x}) = 6\mathbf{x}$, $f_2(\mathbf{y}) = 2\mathbf{y}$
- there exist Non-zero vectors $\mathbf{x}$ and $\mathbf{y}$ such that $f_2(\mathbf{x}) = 2\mathbf{x}$, $f_2(\mathbf{y}) = \mathbf{y}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
there exist Non-zero vectors $\mathbf{x}$ and $\mathbf{y}$ such that $f_2(\mathbf{x}) = 6\mathbf{x}$, $f_2(\mathbf{y}) = 2\mathbf{y}$

4) Consider the two statements given below. *1 point*
(I) There exists a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ satisfying $T(1,1,1) = (1,2,3,0)$, $T(1,2,-1) = (2,1,3,0)$ and $T(1,5,-7) = (0,0,0,1)$.
(II) There exists a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ satisfying $T(2,3,0) = (1,2,3,0)$, $T(1,-1,5) = (2,1,3,0)$ and $T(3,8,-7) = (0,0,0,1)$

- Statement I is TRUE but Statement II is FALSE
- Statement I is TRUE but Statement II is FALSE
- Both Statements I and II are FALSE
- Both Statements I and II are TRUE

No, the answer is incorrect.
Score: 0
Accepted Answers:
Both Statements I and II are FALSE

5) Let $f_1, f_2 : \mathbb{R}^3 \to \mathbb{R}^4$ be linear transformations with *1 point*

$$f_1(\mathbf{e}_1) = f_2(\mathbf{e}_1) = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}, f_1(\mathbf{e}_2) = f_2(\mathbf{e}_2) = \begin{bmatrix}1\\1\\1\\0\end{bmatrix} \text{ and } f_1(\mathbf{e}_3) = \begin{bmatrix}1\\1\\1\\0\end{bmatrix}$$

whereas $f_2(\mathbf{e}_3) = \begin{bmatrix}1\\0\\1\\1\end{bmatrix}$. Then

- both f_1 and f_2 are one-one
- f_1 is one but NOT onto
- f_2 is neither one-one nor onto
- f_2 is one-one but NOT onto

No, the answer is incorrect.
Score: 0
Accepted Answers:
f_2 is one-one but NOT onto

6) Define $D : \mathbb{R}[x;n] \to \mathbb{R}[x;n]$ by $D(f(x)) = f'(x)$, the differentiation with respect to x. Also, let *1 point*
$\mathbb{V} = \{f(x) \in \mathbb{R}[x;n] : f(x) \text{ is a constant polynomial}\}$.
Then

- $\text{Ker}(D) = \mathbb{V}$ and $\text{Rng}(D) = \{f(x) \in \mathbb{R}[x;n] : f(x) \in \mathbb{R}[x;n-1]\}$
- $\text{Ker}(D) = \mathbf{0}$ but $\text{Rng}(D) \neq \{f(x) \in \mathbb{R}[x;n] : f(x) \in \mathbb{R}[x;n-1]\}$
- $\text{Ker}(D) = \mathbb{V}$ but $\text{Rng}(D) \neq \{f(x) \in \mathbb{R}[x;n] : f(x) \in \mathbb{R}[x;n-1]\}$
- $\text{Ker}(D) = \mathbf{0}$ and $\text{Rng}(D) = \{f(x) \in \mathbb{R}[x;n] : f(x) \in \mathbb{R}[x;n-1]\}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\text{Ker}(D) = \mathbb{V}$ *and* $\text{Rng}(D) = \{f(x) \in \mathbb{R}[x;n] : f(x) \in \mathbb{R}[x;n-1]\}$

7) Define $T \in \mathcal{L}(\mathbb{R}^3)$ by $T(\mathbf{e}_1) = \mathbf{e}_1 + \mathbf{e}_3$, $T(\mathbf{e}_2) = \mathbf{e}_2 + \mathbf{e}_3$ and $T(\mathbf{e}_3) = -\mathbf{e}_3$. Then *1 point*

- $T((x,y,z)^T) = (x, y, x-z)^T$, for all $x, y, z \in \mathbb{R}$
- $\text{Ker}(T) \neq \{\mathbf{0}\}$
- $T^2 = \text{Id}$
- $\text{Rng}(T) \neq \mathbb{R}^3$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$T^2 = \text{Id}$

8) Suppose $T : \mathbb{R}^3 \to \mathbb{R}^n$ is a function satisfying $T((1,1,-2)^T) = \mathbf{x}$, $T((-1,2,3)^T) = \mathbf{y}$ and $T((1,10,1)^T) = \mathbf{z}$. *1 point*

- If $\mathbf{z} = 4\mathbf{x} + \mathbf{y}$ then there exists a choice of n for which T is a linear transformation
- If $\mathbf{z} = 4\mathbf{x} + 3\mathbf{y}$ then there exists a choice of n for which T is a linear transformation
- If $\mathbf{z} = 4\mathbf{x} - \mathbf{y}$ then there exists a choice of n for which T is a linear transformation
- There is Neither a choice for n nor a choice for $c, d \in \mathbb{R}$ such that T is a linear transformation and $\mathbf{z} = c\mathbf{x} + d\mathbf{y}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $\mathbf{z} = 4\mathbf{x} + 3\mathbf{y}$ then there exists a choice of n for which T is a linear transformation

9) Let $U = \left\{M = \frac{1}{2}\begin{bmatrix}\sqrt{3} & -1\\1 & \sqrt{3}\end{bmatrix}, N = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\right\}$. For each $A \in U$, define a linear transformation $T_A \in \mathcal{L}(\mathbb{R}^2)$ by $T_A(\mathbf{x}) = A\mathbf{x}$. *1 point*
Then

- T_M represents a clockwise rotation through an angle $\frac{\pi}{3}$
- T_M represents a clockwise rotation through an angle $\frac{\pi}{6}$
- T_N represents a clockwise rotation through an angle $\frac{\pi}{6}$
- T_N represents a counter-clockwise rotation through an angle $\frac{\pi}{4}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
T_N represents a counter-clockwise rotation through an angle $\frac{\pi}{4}$

10) Let $W = \left\{M = \frac{1}{2}\begin{bmatrix}\sqrt{3} & 1\\1 & -\sqrt{3}\end{bmatrix}, N = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\right\}$. For each $A \in W$, define a linear transformation $T_A \in \mathcal{L}(\mathbb{R}^2)$ by $T_A(\mathbf{x}) = A\mathbf{x}$. *1 point*
Then

- T_N represents a reflection through an angle $\frac{\pi}{2}$
- T_M represents a reflection through an angle $\frac{\pi}{12}$
- T_M represents a counter-clockwise rotation through an angle $\frac{\pi}{6}$
- T_N represents a reflection through an angle $\frac{\pi}{4}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
T_M represents a reflection through an angle $\frac{\pi}{12}$