

Unit 8 - Week 6

Course outline

How does an NPTEL online course work?

Week 0: Pre-requisite Assignment

Week 1

Week 2

Week 3

Week 4

Week 5

Week 6

● Definition and Examples of Linear Transformations

○ Results on Linear Transformations

○ Rank-Nullity Theorem and Applications

○ Isomorphism of Vector Spaces

● Ordered Basis of a Finite Dimensional Vector Space

○ Lecture Notes-6

○ Activity Question-6

○ Quiz : Assignment 6

○ Feedback For Week 6

○ Assignment 6 Solution

Week 7

Week 8

Week 9

Week 10

Week 11

Week 12

Live session

VIDEO DOWNLOAD

Assignment 6

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment.

Due on 2020-10-28, 23:59 IST.

Let $A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$, with $a_{ij} \in \mathbb{R}$. Then the 4 fundamental subspaces are:

$$\begin{aligned} 1. \text{ The column space of } A: \\ \text{Col}(A) &= \{Ax : x \in \mathbb{R}^n\} = \text{LS}(A[:, 1], \dots, A[:, n]) \\ &= \text{LS} \left(\left\{ \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}, \dots, \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix} \right\} \right) \end{aligned}$$

$$2. \text{ The column space of } A^T: \text{Col}(A^T) = \text{LS}(A^T[:, 1], \dots, A^T[:, m]) = \{A^T x : x \in \mathbb{R}^m\}.$$

$$3. \text{ The null space of } A: \text{Null}(A) = \mathcal{N}(A) = \{x \in \mathbb{R}^n : Ax = \mathbf{0}\}.$$

$$4. \text{ The null space of } A^T: \text{Null}(A^T) = \mathcal{N}(A^T) = \{x \in \mathbb{R}^m : A^T x = \mathbf{0}\}.$$

1) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 2 & 6 & 8 \\ 2 & 8 & 10 \end{bmatrix}$. If $\text{Null}(A) = \text{LS} \left(\begin{bmatrix} 1 \\ \alpha \\ -1 \end{bmatrix} \right)$ then the value of α equals

No, the answer is incorrect. Score: 0
Accepted Answers: (Type: Range) 0.9, 1.1

1 point

2) There exists $M \in M_{3,4}(\mathbb{R})$ such that $\text{LS}([1 \ 1 \ 1 \ 1]^T) = \text{Col}(M)$ and $\text{Null}(M) = \text{LS}([1 \ 1 \ 1 \ 1]^T)$.

- TRUE
 FALSE

1 point

No, the answer is incorrect. Score: 0
Accepted Answers: FALSE

3) There exists $M \in M_{3,4}(\mathbb{R})$ such that $\text{LS}([1 \ 1 \ 1 \ 1]^T, [1 \ 1 \ -1 \ 1]^T) = \text{Col}(M)$ and

1 point

$$\text{Null}(M) = \text{LS}([1 \ 1 \ 1 \ 1]^T).$$

- TRUE
 FALSE

No, the answer is incorrect. Score: 0
Accepted Answers: FALSE

4) There exists $M \in M_{3,4}(\mathbb{R})$ such that $\text{LS}([1 \ 1 \ 0 \ -2]^T, [1 \ 1 \ -1 \ -1]^T) = \text{Col}(M^T)$ and $\text{Null}(M) = \text{LS}([1 \ 1 \ 1 \ 1]^T)$.

1 point

- TRUE
 FALSE

No, the answer is incorrect. Score: 0
Accepted Answers: FALSE

5) There exists $M \in M_{3,4}(\mathbb{R})$ such that $\text{LS}([1 \ 1 \ 0 \ -2]^T, [1 \ 1 \ -1 \ -1]^T) = \text{Col}(M^T)$ and $\text{Null}(M) = \text{LS}([1 \ 1 \ 1 \ 1]^T, [-1 \ 1 \ 0 \ 0]^T)$.

1 point

- TRUE
 FALSE

No, the answer is incorrect. Score: 0
Accepted Answers: TRUE

6) Does there exists a matrix whose Column space contains $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and row space contains

1 point

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}?$$

- TRUE
 FALSE

No, the answer is incorrect. Score: 0
Accepted Answers: TRUE

7) Does there exists a matrix whose Column space has basis $\left\{ \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \right\}$, null-space has basis $\left\{ \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \right\}$?

1 point

- TRUE
 FALSE

No, the answer is incorrect. Score: 0
Accepted Answers: FALSE

8) Does there exists a matrix whose Column space has basis $\left\{ \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \right\}$, null-space contains $\left\{ \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \right\}$?

1 point

- TRUE
 FALSE

No, the answer is incorrect. Score: 0
Accepted Answers: TRUE

9) Does there exists a matrix whose dimension of null-space is one more than the dimension of left null-space?

1 point

- TRUE
 FALSE

No, the answer is incorrect. Score: 0
Accepted Answers: TRUE

10) Does there exists a matrix whose Left null-space contains $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and row space contains $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$?

1 point

- TRUE
 FALSE

No, the answer is incorrect. Score: 0
Accepted Answers: TRUE