Unit 7 - Week 5

Assignment 5

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-10-21, 23:59 IST.

Let $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$, with $a_{ij} \in \mathbb{R}$.

Then the 4 fundamental subspaces are:

1. The column space of A:
$$\mathrm{Col}(A) = \{A\mathbf{x} : \mathbf{x} \in \mathbb{R}^n\} = \mathrm{LS}(A[:,1], \ldots, A[:,n]) = \mathrm{LS}\left(\left\{\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}, \ldots, \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}\right\}\right)$$
2. The column space of A^T:
$\mathrm{Col}(A^T) = \mathrm{LS}(A^T[:,1], \ldots, A^T[:,m]) = \{A^T\mathbf{x} : \mathbf{x} \in \mathbb{R}^m\}$.
3. The null space of A:
$\mathrm{Null}(A) = \mathcal{N}(A) = \{\mathbf{x} \in \mathbb{R}^n : A\mathbf{x} = \mathbf{0}\}$.
4. The null space of A^T:
$\mathrm{Null}(A^T) = \mathcal{N}(A^T) = \{\mathbf{x} \in \mathbb{R}^m : A^T\mathbf{x} = \mathbf{0}\}$.

1) Consider the sets *1 point*
$$S = \{(1,0,0,0), (1,1,0,0), (1,2,0,0), (1,1,1,1)\}$$
and
$$T = \{(1,0,2,1), (1,3,2,1), (4,1,2,2)\}.$$
Then

- S is linearly independent but T is linearly dependent subset of $\mathbb{R}^4$
- both S and T are linearly dependent subset of $\mathbb{R}^4$
- both S and T are linearly independent subset of $\mathbb{R}^4$
- S is linearly dependent but T is linearly independent subset of $\mathbb{R}^4$

No, the answer is incorrect.
Score: 0
Accepted Answers:
S is linearly dependent but T is linearly independent subset of $\mathbb{R}^4$

2) The number of vectors in a maximal linearly independent subset of
$$S = \left\{\begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}\right\}.$$
equals ....

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 4.9,5.1

1 point

3) Let $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ be a subset of a vector space $\mathbb{V}$ over $\mathbb{F}$. Suppose $LS(S) = \mathbb{V}$. If S is a linearly dependent set then *1 point*

- S is a basis of $\mathbb{V}$ over $\mathbb{F}$
- each $\mathbf{v} \in \mathbb{V}$ is uniquely expressible as a linear combination of vectors from S
- it is possible to get a proper subset T of S such that T is a basis of $\mathbb{V}$ over $\mathbb{F}$
- $\dim(\mathbb{V}) = p$

No, the answer is incorrect.
Score: 0
Accepted Answers:
it is possible to get a proper subset T of S such that T is a basis of $\mathbb{V}$ over $\mathbb{F}$

4) For $A \in \mathbb{M}_k(\mathbb{R})$, let $S = \{\mathbf{u}_1, \ldots, \mathbf{u}_k\} \subset \mathbb{V}$ and $T = \left\{\sum_{i=1}^{k} a_{i1}\mathbf{u}_i, \ldots, \sum_{i=1}^{k} a_{ik}\mathbf{u}_i\right\}$. *1 point*

- Then S is linearly independent implies T is linearly independent
- Then T is linearly independent implies S is linearly independent
- If A is invertible then T is linearly independent
- Then T is linearly independent implies A need NOT be invertible

No, the answer is incorrect.
Score: 0
Accepted Answers:
Then T is linearly independent implies S is linearly independent

5) Let $S_1 = \{\mathbf{u}_1, \ldots, \mathbf{u}_n\}$ and $S_2 = \{\mathbf{w}_1, \ldots, \mathbf{w}_n\}$ be subsets of a real vector space $\mathbb{V}$. Also, let $[\mathbf{w}_1 \ \cdots \ \mathbf{w}_n] = [\mathbf{u}_1 \ \cdots \ \mathbf{u}_n]A$ for some matrix $A \in \mathbb{M}_n(\mathbb{R})$. *1 point*

- Then S_1 is linearly independent implies S_2 is linearly independent
- Then S_2 is linearly independent implies S_1 is linearly independent
- If A is invertible then S_2 is linearly independent
- Then S_2 is linearly independent implies A need NOT be invertible

No, the answer is incorrect.
Score: 0
Accepted Answers:
Then S_2 is linearly independent implies S_1 is linearly independent

6) Consider $\mathbb{W} = \{\mathbf{v} \in \mathbb{R}^6 : v_1 + v_2 + v_3 = 0, v_2 + v_3 + v_4 = 0, v_5 + v_6 = 0\}$. *0 points*

If $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{u}_4 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{u}_5 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and

$\mathbf{u}_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$ then a basis of $\mathbb{W}$ equals

- $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_5\}$
- $\{\mathbf{u}_1, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5\}$
- $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_5, \mathbf{u}_6\}$
- $\{\mathbf{u}_1, \mathbf{u}_3, \mathbf{u}_5, \mathbf{u}_6\}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\{\mathbf{u}_1, \mathbf{u}_3, \mathbf{u}_5, \mathbf{u}_6\}$

7) Let $\mathbb{V} = \{(x, y, z, u) \in \mathbb{R}^4 \mid x - y - z = 0, x + z - u = 0\}$ and $\mathbb{W} = \{(x, y, z, w)^T \in \mathbb{R}^4 \mid x + y - z + w = 0\}$ be two subspaces of $\mathbb{R}^4$. Then *1 point*

- $\{[-1,-2,1,0]^T, [1,1,0,1]^T, [1,0,1,-2]^T\}$ is a basis of $\mathbb{V}$
- $\{[-1,-2,1,0]^T, [1,1,0,1]^T, 1,0,1,-2]^T\}$ is a basis of $\mathbb{W}$
- $\{[-1,1,0,0]^T, [1,0,1,0]^T, [-1,0,0,1]^T\}$ is a basis of $\mathbb{V}$
- $\{[-1,1,0,0]^T, [1,0,1,0]^T, [-1,0,0,1]^T\}$ is a basis of $\mathbb{W}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\{[-1,1,0,0]^T, [1,0,1,0]^T, [-1,0,0,1]^T\}$ *is a basis of* $\mathbb{W}$

8) Let $\mathbb{V} = \{A \in \mathbb{M}_n(\mathbb{R}) \mid A = A^T\}$ and $\mathbb{W} = \{A \in \mathbb{M}_n(\mathbb{R}) \mid A^T = -A\}$. Then $\mathbb{V}$ and $S = \{\mathbf{e}_{ij} + \mathbf{e}_{ji} \mid 1 \le i < j \le n\}$, $T = \{\mathbf{e}_{ij} - \mathbf{e}_{ji} \mid 1 \le i < j \le n\}$ and $U = \{\mathbf{e}_{ii} | 1 \le i \le n\}$. *1 point*

- S is a basis of $\mathbb{V}$ and T is a basis of $\mathbb{W}$
- $S \cup U$ is a basis of $\mathbb{V}$ and $T \cup U$ is a basis of $\mathbb{W}$
- $S \cup U$ is a basis of $\mathbb{V}$ and T is a basis of $\mathbb{W}$
- S is a basis of $\mathbb{V}$ and $T \cup U$ is a basis of $\mathbb{W}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$S \cup U$ is a basis of $\mathbb{V}$ and T is a basis of $\mathbb{W}$

9) Let $A = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, $\mathrm{Null}(A) = \{\mathbf{x} : A\mathbf{x} = \mathbf{0}\}$ and $\mathrm{Row}(A) = \{A^T\mathbf{y} : \mathbf{y} \in \mathbb{R}^3\}$. *1 point*

Also, let
$\mathbb{W} = \{p(x) \in \mathbb{R}[x;4] \mid p(-1) = p(1) = 0\}$ be a subspace of $\mathbb{R}[x;4]$. Then

- $\dim(\mathrm{Null}(A)) = 2$, $\dim(\mathrm{Row}(A)) = 2$ and $\dim(\mathbb{W}) = 2$
- $\dim(\mathrm{Null}(A)) = 2$, $\dim(\mathrm{Row}(A)) = 3$ and $\dim(\mathbb{W}) = 3$
- $\dim(\mathrm{Null}(A)) = 3$, $\dim(\mathrm{Row}(A)) = 2$ and $\dim(\mathbb{W}) = 3$
- $\dim(\mathrm{Null}(A)) = 2$, $\dim(\mathrm{Row}(A)) = 3$ and $\dim(\mathbb{W}) = 2$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\dim(\mathrm{Null}(A)) = 2$, $\dim(\mathrm{Row}(A)) = 3$ *and* $\dim(\mathbb{W}) = 3$

10) Let $\mathbb{V}$ be a vector space with basis $S = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_n\}$. Let T be a subset of $\mathbb{V}$ having m vectors. Now, consider the two statements given below: *1 point*

(I) If $m > n$ then T is linear dependent.

(ii) If $m < n$ then T does NOT span $\mathbb{V}$.

Which among the following is a CORRECT Option?

- (I) is TRUE but (II) is FALSE
- Both (I) and (II) are FALSE
- (I) is FALSE but (II) is TRUE
- Both (I) and (II) are TRUE

No, the answer is incorrect.
Score: 0
Accepted Answers:
Both (I) and (II) are TRUE