

## Unit 5 - Week 3

## Course outline

How does an NPTEL online course work?

Week 0: Pre-requisite Assignment

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Week 2

Week 3

Rank of a matrix

Solution Set of a System of Linear Equations

System of n Linear Equations in n Unknowns

Determinant

Permutations and the Inverse of a Matrix

Inverse and the Cramer's Rule

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VIDEO DOWNLOAD

## Assignment 3

The due date for submitting this assignment has passed.  
As per our records you have not submitted this assignment.

**Due on 2020-10-07, 23:59 IST.**

For Questions (1) to (4), consider the linear system  $M\mathbf{x} = \mathbf{b}$  where  $M \in M_4(\mathbb{R})$  and  $\mathbf{b}^T = [1, 2, -1, -2]$ .

1) If  $M\mathbf{x} = \mathbf{b}$  has NO solution then  $M$  is invertible. 1 point

- TRUE  
 FALSE

No, the answer is incorrect.

Score: 0

Accepted Answers:

FALSE

2) If  $M\mathbf{x} = \mathbf{b}$  has NO solution then the homogeneous system  $M\mathbf{x} = \mathbf{0}$  has only the trivial solution. 1 point

- TRUE  
 FALSE

No, the answer is incorrect.

Score: 0

Accepted Answers:

FALSE

3) Let  $\mathbf{c}^T = [-1, -2, 1, 2]$ . If  $M\mathbf{x} = \mathbf{b}$  has NO solution then the system  $M\mathbf{x} = \mathbf{c}$  has no solution. 1 point

- TRUE  
 FALSE

No, the answer is incorrect.

Score: 0

Accepted Answers:

TRUE

4) Let  $R = \text{RREF}(M)$ . If  $M\mathbf{x} = \mathbf{b}$  has NO solution then  $R[3, :] = [0, 1, *, *]$  1 point

- TRUE  
 FALSE

No, the answer is incorrect.

Score: 0

Accepted Answers:

FALSE

5) 1 point

$$\text{Let } M = \begin{bmatrix} 1 & 1 & 4 & 5 \\ 3 & 2 & 1 & 7 \\ 7 & 8 & 9 & 3 \\ 5 & 7 & 1 & 1 \end{bmatrix} \text{ and } N = \begin{bmatrix} 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ 1 & x_3 & x_3^2 & x_3^3 \\ 1 & x_4 & x_4^2 & x_4^3 \end{bmatrix}. \text{ Then}$$

- 11 divides  $\det(M)$  and  $\det(N) = \prod_{i < j} (x_j - x_i)$   
 13 divides  $\det(M)$  and  $\det(N) = \prod_{i < j} (x_j - x_i)$   
 11 divides  $\det(M)$  and  $\det(N) = \prod_{i < j} (x_i - x_j)$   
 13 divides  $\det(M)$  and  $\det(N) = \prod_{i < j} (x_i - x_j)$

No, the answer is incorrect.

Score: 0

Accepted Answers:

11 divides  $\det(M)$  and  $\det(N) = \prod_{i < j} (x_j - x_i)$

6) For  $M \in M_n(\mathbb{R})$ , consider the two statements given below. 1 point

(a) The system  $M\mathbf{x} = \mathbf{b}$  has a solution for every  $\mathbf{b}$ .

(b) The system  $M\mathbf{x} = \mathbf{0}$  has a non-trivial solution. Then,

- there exists a choice for  $M$  and  $\mathbf{b}$  such that both the statements are TRUE simultaneously.  
 there does NOT exist  $M$  for which both the statements can hold TRUE simultaneously.

No, the answer is incorrect.

Score: 0

Accepted Answers:

there does NOT exist  $M$  for which both the statements can hold TRUE simultaneously.

7) 1 point

$$\text{Let } M = \begin{bmatrix} 1 & 2 & 3 & 4 & \dots & n \\ 2 & 2 & 3 & 4 & \dots & n \\ 3 & 3 & 3 & 4 & \dots & n \\ \dots & \dots & \dots & \dots & \dots & n \\ n & n & n & n & \dots & n \end{bmatrix} \text{ and } N = \begin{bmatrix} 1^2 & 3^2 & 5^2 & 8^2 \\ 9^2 & 11^2 & 13^2 & 16^2 \\ 20^2 & 22^2 & 24^2 & 27^2 \\ 31^2 & 33^2 & 35^2 & 38^2 \end{bmatrix}. \text{ Then}$$

- $\det(M) = (-1)^n n$  and  $\det(N) = 0$   
  $\det(M) = (-1)^{n+1} n$  and  $\det(N) = 0$   
  $\det(M) = (-1)^n n$  and  $\det(N) \neq 0$   
  $\det(M) = (-1)^{n+1} n$  and  $\det(N) \neq 0$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$\det(M) = (-1)^{n+1} n$  and  $\det(N) = 0$

8) Let  $M = [m_{ij}]$  and  $B = [p^{i-j} m_{ij}]$ , for some  $p \neq 0$ . Also, let  $N \in M_4(\mathbb{R})$  be a matrix having all it's rows as  $[a \ b \ c \ d]$ . Then 1 point

- $\det(B) = \det(M)$  and  $\det(I + N) = 1 + a + b + c + d$   
  $\det(B) \neq \det(M)$  but  $\det(I + N) = 1 + a + b + c + d$   
  $\det(B) = \det(M)$  and  $\det(I + N) = 1$   
  $\det(B) \neq \det(M)$  but  $\det(I + N) = 1$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$\det(B) = \det(M)$  and  $\det(I + N) = 1 + a + b + c + d$

9) The NEGATIVE value of  $\lambda$  for which the system  $\lambda x + 3y = 0$ ,  $(\lambda + 6)y = 0$ , in the unknowns  $x$  and  $y$ , has a non-trivial solution equals .....

No, the answer is incorrect.

Score: 0

Accepted Answers:

(Type: Range) -6.1, -5.9

10) Let  $M \in M_2(\mathbb{R})$  be an orthogonal matrix ( $M^T M = M M^T = I_2$ ). Then, which among the following is TRUE? 1 point

- If  $\det(M) = 1$  then  $M = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ , for some  $\theta \in (-\pi, \pi]$ , and  $M$  represents a reflection about a line passing through origin  
 If  $\det(M) = -1$  then  $M = \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{bmatrix}$ , for some  $\theta \in (-\pi, \pi]$  and  $M$  represents a counter-clockwise rotation through the angle  $\theta$   
 If  $\det(M) = 1$  then  $M = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ , for some  $\theta \in (-\pi, \pi]$ , and  $M$  represents a counter-clockwise rotation through the angle  $-\theta$   
 Fix  $n \geq 3$ . Then, there exists orthogonal matrices  $M \in M_n(\mathbb{R})$  such that  $\det(M) \neq \pm 1$

No, the answer is incorrect.

Score: 0

Accepted Answers:

If  $\det(M) = 1$  then  $M = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ , for some  $\theta \in (-\pi, \pi]$ , and  $M$  represents a counter-clockwise rotation through the angle  $-\theta$