Unit 12 - Week 10

Assignment 10

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-11-25, 23:59 IST.

1) Let $M = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$ and $N = \begin{bmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{bmatrix}$. Then, which among the following is an INCORRECT Option? *1 point*

- The eigen-pairs of M are $\left(3, \begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$ and $\left(-1, \begin{bmatrix} -1 \\ 2 \end{bmatrix}\right)$
- The eigenvalues of N are $0, -2$ and -3
- The eigenvector of N corresponding to -2 equals $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$
- The eigenvector of N corresponding to -3 equals $\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
The eigenvector of N corresponding to -3 equals $\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

2) Let $M = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $N = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$. Then, which among the following is an INCORRECT Option? *1 point*

- The eigen-pairs of M are $\left(\cos\theta + i\sin\theta, \begin{bmatrix} 1 \\ -i \end{bmatrix}\right)$ and $\left(\cos\theta - i\sin\theta, \begin{bmatrix} 1 \\ i \end{bmatrix}\right)$
- The eigen-pairs of M are $\left(\cos\theta + i\sin\theta, \begin{bmatrix} 1 \\ i \end{bmatrix}\right)$ and $\left(\cos\theta - i\sin\theta, \begin{bmatrix} 1 \\ -i \end{bmatrix}\right)$
- The eigen-pairs of N are $\left(1, \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix}\right)$ and $\left(-1, \begin{bmatrix} \sin(\theta/2) \\ -\cos(\theta/2) \end{bmatrix}\right)$
- The eigen-pairs of N are $\left(1, \begin{bmatrix} 1+\cos\theta \\ \sin\theta \end{bmatrix}\right)$ and $\left(-1, \begin{bmatrix} 1-\cos\theta \\ -\sin\theta \end{bmatrix}\right)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
The eigen-pairs of M are $\left(\cos\theta + i\sin\theta, \begin{bmatrix} 1 \\ i \end{bmatrix}\right)$ *and* $\left(\cos\theta - i\sin\theta, \begin{bmatrix} 1 \\ -i \end{bmatrix}\right)$

3) Let $M = [m_{ij}] \in \mathbb{M}_n(\mathbb{C})$ Then, which among the following is an INCORRECT Option? *1 point*

- If $\sum_{j=1}^{n} m_{ij} = a$, for all $1 \le i \le n$ then a is an eigenvalue of M
- M and M^T have the same set of eigenvalues
- M and M^* have the same set of eigenvalues
- The eigenvectors of M and M^T need NOT be the same

No, the answer is incorrect.
Score: 0
Accepted Answers:
M and M^* have the same set of eigenvalues

4) Let $B \in \mathbb{M}_n(\mathbb{C})$ and $C \in \mathbb{M}_m(\mathbb{C})$. Now, define $S = \begin{bmatrix} B & \mathbf{0} \\ \mathbf{0} & C \end{bmatrix}$. Then, which among the following is an INCORRECT Option? *1 point*

- If $(\alpha, \mathbf{x})$ is an eigen-pair for B then $\left(\alpha, \begin{bmatrix} \mathbf{x} \\ \mathbf{0} \end{bmatrix}\right)$ is an eigen-pair for S
- If $(\beta, \mathbf{y})$ is an eigen-pair for C then $\left(\beta, \begin{bmatrix} \mathbf{0} \\ \mathbf{y} \end{bmatrix}\right)$ is an eigen-pair for S
- If $\left(\lambda, \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}\right)$ is an eigen-pair for S with $\mathbf{x} \neq \mathbf{0}$ and $\mathbf{y} \neq \mathbf{0}$ then $(\lambda, \mathbf{x})$ is an eigen-pair of B and $(\lambda, \mathbf{y})$ is an eigen-pair of C
- If $\left(\lambda, \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}\right)$ is an eigen-pair for S then one of $\mathbf{x}$ or $\mathbf{y}$ must be the zero vector

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $\left(\lambda, \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}\right)$ is an eigen-pair for S then one of $\mathbf{x}$ or $\mathbf{y}$ must be the zero vector

5) Let $M = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$. If $\begin{bmatrix} 1 \\ b \\ c \end{bmatrix}$ is an eigenvector of M corresponding to the largest eigenvalue then the value of $b^2 + c^2$ equals .... *1 point*

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 3.9,4.1

6) Let $M = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$. If $\begin{bmatrix} a \\ 1 \\ c \end{bmatrix}$ is an eigenvector of M corresponding to the largest eigenvalue then the value of $a^2 + c^2$ equals .... *1 point*

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.9,1.1

1 point

7) Let M be an $n \times n$ matrix. Then, which among the following in an INCORRECT Option? *1 point*

- If M is idempotent then eigenvalues of M are either 0 or 1.
- If M is nilpotent then all eigenvalues of M are 0
- If $M^* = M$ then, the eigenvalues are all real.
- If $M^* = -M$ and $M \neq \mathbf{0}$ then, the eigenvalues are purely imaginary.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $M^* = -M$ and $M \neq \mathbf{0}$ then, the eigenvalues are purely imaginary.

8) Let M be a 3×3 orthogonal matrix. Then, which among the following is an INCORRECT Option? *1 point*

- If $\det(M) = 1$, then there exists $\mathbf{v} \in \mathbb{R}^3 \setminus \{\mathbf{0}\}$ such that $M\mathbf{v} = \mathbf{v}$
- If $\det(M) = -1$, then there exists $\mathbf{v} \in \mathbb{R}^3 \setminus \{\mathbf{0}\}$ such that $M\mathbf{v} = -\mathbf{v}$
- If M is a rotation then it fixes a line, say ℓ, passing through origin and rotates any vector that lies in the orthogonal complement of ℓ
- If M is a reflection then it is a rotation about a line, say ℓ, by an angleπ

No, the answer is incorrect.
Score: 0
Accepted Answers:
If M is a reflection then it is a rotation about a line, say ℓ, by an angleπ

9) Let $M = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}$. Notice that$\mathbf{x}_1 = \frac{1}{\sqrt{3}}\mathbf{1}$ is an eigenvector for M. Let $\mathbf{x}_2 = (0,1,0)^T$ and $\mathbf{x}_3 = (0,0,1)^T$ and take $\mathcal{B} = (\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_2)$ as an ordered basis. Put $X = [\mathbf{x}_1 \quad \mathbf{x}_2 \quad \mathbf{x}_3]$. *1 point*
If

$$X^{-1}MX = \begin{bmatrix} 6 & \alpha & \beta \\ 0 & 0 & -2 \\ 0 & 1 & -2 \end{bmatrix} \text{ and } N = \begin{bmatrix} 0 & -2 \\ 1 & -2 \end{bmatrix} \text{ then}$$

- both the eigenvalues of N are eigenvalues of M
- one of the eigenvalues of N is an eigenvalue of M
- none of the eigenvalues of N are eigenvalues of M

No, the answer is incorrect.
Score: 0
Accepted Answers:
both the eigenvalues of N are eigenvalues of M

10) Let $M \in \mathbb{M}_{m\times n}(\mathbb{R})$ and $N \in \mathbb{M}_{n\times m}(\mathbb{R})$. Then, which among the following is an INCORRECT Option? *1 point*

- $\alpha \in \sigma(MN) \iff \alpha \in \sigma(NM)$
- If $\alpha \neq 0$ then $\text{Alg.Mul}_\alpha(MN) = \text{Alg.Mul}_\alpha(NM)$ and $\text{Geo.Mul}_\alpha(MN) = \text{Geo.Mul}_\alpha(NM)$
- If $0 \in \sigma(MN)$ and $n = m$ then $\text{Alg.Mul}_0(MN) = \text{Alg.Mul}_0(NM)$as there are n eigenvalues, counted with multiplicity
- $\text{Geo.Mul}_0(MN)$ need NOT equal $\text{Geo.Mul}_0(NM)$ even when$n = m$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\alpha \in \sigma(MN) \iff \alpha \in \sigma(NM)$