Unit 3 - Week 1

Assignment 1

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-09-30, 23:59 IST.

Following is a collection of matrices that are useful for answering Questions 1 to 3.

$$P=\begin{bmatrix}2&3&4\\3&4&5\\4&5&6\end{bmatrix} Q=\begin{bmatrix}2^2&2^3&2^4\\2^3&2^4&2^5\\2^4&2^5&2^6\end{bmatrix} R=\begin{bmatrix}2&1&1\\1&2&1\\1&1&2\end{bmatrix} S=\begin{bmatrix}1&1&0\\1&1&1\\0&1&1\end{bmatrix}$$

$$U=\begin{bmatrix}1/2&1/2&0\\1/2&1/2&0\\0&0&1\end{bmatrix} V=\begin{bmatrix}1&-1&2\\-1&3&5\\2&5&-1\end{bmatrix} W=\begin{bmatrix}0&-1&2\\1&0&5\\-2&-5&0\end{bmatrix} Z=\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}$$

1) For each condition given below, there is a 3×3 matrix in the previous table. Among the Options given below, choose the OPTION which gives a correct matching. *1 point*
(I) $a_{ij}=1$ if $i\neq j$ and 2 otherwise.
(II) $a_{ij}=1$ if $|i-j|\leq 1$ and 0 otherwise.
(III) $a_{ij}=i+j$
(IV) $a_{ij}=2^{i+j}$

- $(I)\to S,(II)\to R,(III)\to U,(IV)\to Q$
- $(I)\to R,(II)\to S,(III)\to P,(IV)\to Q$
- $(I)\to S,(II)\to R,(III)\to Q,(IV)\to Z$
- $(I)\to R,(II)\to S,(III)\to Q,(IV)\to P$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$(I)\to R,(II)\to S,(III)\to P,(IV)\to Q$

2) If M and N satisfy $M^2=\mathbf{0}$ and $N^3=N$ (from the given set of matrices) then *1 point*

- $M\to U, N\to Z$
- $M\to Z, N\to Z$
- $M\to Z, N\to U$
- $M\to Z, N\to Z$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$M\to Z, N\to U$

3) If M and N satisfy $M^T=M$ and $N^T=-N$ (from the given set of matrices) then *1 point*

- $M\in\{P,Q,R,U\},\ \ N\in\{Z,W\}$
- $M\in\{U,V,R,Z\},\ \ N\in\{W\}$
- $M\in\{S,U,V\},\ \ N\in\{Z,W\}$
- $M\in\{P,Q,R,S,U,V\},\ \ N\in\{W\}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$M\in\{P,Q,R,S,U,V\},\ \ N\in\{W\}$

4) Suppose $M=\begin{bmatrix}a&b\\c&d\end{bmatrix}$ satisfies $(I+3M)^{-1}=\begin{bmatrix}1&2\\2&1\end{bmatrix}$. Then *1 point*

- $9a=4$
- $9(a+b)=-2$
- $9c=-4$
- $9a^2=16$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$9(a+b)=-2$

5) Let $M=[m_{ij}]$ be an invertible matrix and $N=[p^{i-j}m_{ij}]$, for some $p\in\mathbb{C}$, $p\neq 0$. *1 point*

- Then $N^{-1}=[p^{i-j}(M^{-1})_{ij}]$
- Then $N^{-1}=[p^{j-i}(M^{-1})_{ij}]$
- Then $N^{-1}=[p^{i+j}(M^{-1})_{ij}]$
- Then $N^{-1}=M^{-1}$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Then $N^{-1}=[p^{i-j}(M^{-1})_{ij}]$

6) Let $J=\mathbf{1}\mathbf{1}^T\in\mathbb{M}_n(\mathbb{R})$. Then each entry of J equals1. Which among the following is an INCORRECT Option? *1 point*

- $J^2=nJ$
- $(\alpha J+\beta I_n)$ and $(\gamma J+\delta I_n)$ commute for any complex numbers α,β,γ and δ
- $bJ+(a-b)I_n$ is invertible whenever $a\neq b$
- $bJ+(a-b)I_n$ is invertible whenever $a\neq b$ and $a+(n-1)b\neq 0$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$bJ+(a-b)I_n$ *is invertible whenever* $a\neq b$

7) Let $M\in\mathbb{M}_n(\mathbb{C})$. Which among the following is an INCORRECT Option? *1 point*

- If $(M\mathbf{x})^*\mathbf{x}=\mathbf{x}^*M^*\mathbf{x}\in\mathbb{R}$ for every$\mathbf{x}\in\mathbb{M}_{n,1}(\mathbb{C})$ then M is a Hermitian matrix.
- If M is skew-Hermitian matrix then the diagonal entries of A are essentially zero
- If M is skew-Hermitian matrix then the diagonal entries of A are either zero or purely imaginary.
- If $M\mathbf{x}=\mathbf{0}$ for all $\mathbf{x}\in\mathbb{M}_{n,1}(\mathbb{C})$ then $M=\mathbf{0}$, the zero matrix.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If M is skew-Hermitian matrix then the diagonal entries of A are essentially zero

8) If $M\in\mathbb{M}_n(\mathbb{C})$ is an invertible matrix then, which among the following is an INCORRECT Option? Then *1 point*

- $M[:,j]\neq\mathbf{0}$, for any j
- $M[i,:]\neq M[j,:]$, for any i and j
- $M[:,i]\neq M[:,j]$, for any i and j
- there may exist columns i,j and k of M and a choice of $\alpha,\beta\in\mathbb{C}$ such that $M[:,k]=\alpha M[:,i]+\beta M[:,j]$, whenever $n\geq 3$

No, the answer is incorrect.
Score: 0
Accepted Answers:
there may exist columns i,j and k of M and a choice of $\alpha,\beta\in\mathbb{C}$ such that
$M[:,k]=\alpha M[:,i]+\beta M[:,j]$, *whenever* $n\geq 3$

9) For $M=[m_{ij}]\in\mathbb{M}_n(\mathbb{C})$, define the trace of M, denoted $\mathrm{tr}(M)$, by$\mathrm{tr}(M)=\sum_{i=1}^{n}m_{ii}$. Which among the following is an INCORRECT Option? *1 point*

- For each $M\in\mathbb{M}_n(\mathbb{C})$ there exists $N\in\mathbb{M}_n(\mathbb{C})$ such that $\mathrm{tr}(MN)=\mathrm{tr}(NM)$
- If M is invertible then $\mathrm{tr}(MNM^{-1})=\mathrm{tr}(N)$, for all $N\in\mathbb{M}_n(\mathbb{C})$.
- There exists matrices M and N such that $MN-NM=cI_n$, for some $c\neq 0$.
- If $N=[n_{ij}]\in\mathbb{M}_n(\mathbb{C})$ then $\mathrm{tr}(M^TN)=\sum_{i=1}^{n}\sum_{j=1}^{n}m_{ij}n_{ij}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
There exists matrices M and N such that $MN-NM=cI_n$, for some $c\neq 0$.

10) For $1\leq k\leq m$ and $1\leq \ell\leq n$, define $\mathbf{e}_{k\ell}\in\mathbb{M}_{m,n}(\mathbb{C})$ by *1 point*
$(\mathbf{e}_{k\ell})_{ij}=\begin{cases}1, & \text{if } (k,\ell)=(i,j)\\ 0, & \text{otherwise.}\end{cases}\ 1\leq k\leq m$
Then, the matrices $\mathbf{e}_{k\ell}$ for $1\leq k\leq m$ and $1\leq\ell\leq n$ are called the {\bf standard basis} elements for $\mathbb{M}_{m,n}(\mathbb{C})$.
So, if $\mathbf{e}_{k\ell}\in\mathbb{M}_{2,3}(\mathbb{C})$ then $\mathbf{e}_{11}=\begin{bmatrix}1&0&0\\0&0&0\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1&0&0\end{bmatrix}$,
$\mathbf{e}_{12}=\begin{bmatrix}0&1&0\\0&0&0\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0&1&0\end{bmatrix}$ and $\mathbf{e}_{22}=\begin{bmatrix}0&0&0\\0&1&0\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0&1&0\end{bmatrix}$.
In particular, if $\mathbf{e}_{ij}\in\mathbb{M}_n(\mathbb{C})$ then $\mathbf{e}_{ij}=\mathbf{e}_i\mathbf{e}_j^T=\mathbf{e}_i\mathbf{e}_j^*$, for $1\leq i,j\leq n$.
Which among the following is an INCORRECT Option?

- $\mathbf{e}_{12}\mathbf{e}_{11}=\mathbf{0}$
- $\mathbf{e}_{11}\mathbf{e}_{12}=\mathbf{e}_{12}$
- $\mathbf{e}_{12}\mathbf{e}_{22}=\mathbf{e}_{12}$
- $\mathbf{e}_{22}\mathbf{e}_{12}=\mathbf{e}_{21}$$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mathbf{e}_{22}\mathbf{e}_{12}=\mathbf{e}_{21}$$