Assignment - 4

Curves AND surfaces **August 09, 2016**

Total Marks: 30

1. The Second Fundamental Form of a surface patch σ is zero every where. Prove that σ is a part of a plane. [6]

2. A unit speed curve $(f(u), 0, g(u))$ is rotated around z-axis to generate a surface. Calculate the geodesic curvatures of
   (a) a meridian: $v = v_0$. [3]
   (b) a parallel: $u = u_0$. [3]

3. Show that the principal curvatures of a surface are the maximum and minimum values of the normal curvature of all curves on the surface that pass through the point. Also show that principal vectors are the tangent vectors of the curve that gives these maximum and minimum value. (Hint: Use Euler's Theorem). [6]

4. Consider the surface of revolution $\sigma(u, v) = (f(u)\cos v, f(u)\sin v, g(u))$. Let K_G denote the Gaussian Curvature of σ. Describe σ in the following cases:
   (a) $K_G = 0$ everywhere. [3]
   (b) $K_G = 1$ everywhere. [3]

5. Calculate the FFF of the pseudosphere under the reparamettrization $U = \frac{v^2+w^2-1}{v^2+(w+1)^2}$, $V = \frac{-2v}{v^2+(w+1)^2}$, where $w = e^{-u}$. [6]