

Assignment - 2

Curves AND surfaces

July 26, 2016

Total Marks: 30

1. Show that the sphere $S^2 = \{(x, y, z) : x^2 + y^2 + z^2 = 1\}$ can not be covered by a single surface patch. [5]

2. Verify that following are regular smooth surfaces:
(i) plane, (ii) sphere, (iii) level surfaces of a smooth function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$, (iv) ellipsoid, (v) torus. [10]

3. Show that the surface $S = \{(x, y, z) : z = x^2 - y^2, z > 0\}$ (called hyperbolic paraboloid) can be parametrized as $\sigma(r, \theta) = (r \cosh \theta, r \sinh \theta, r^2)$. Find the open set $U \subset \mathbb{R}^2$ where σ is defined. [5]

4. Find the equation of the tangent plane of the surface in Exercise 3 at the point $(1, 0, 1)$. [5]

5. Show that tangent plane of a regular smooth surface is invariant under reparametrization. [5]