# Question Bank-1

1. What is necessary condition for a function to having local minima? Give an example where necessary condition is not sufficient.

2. Prove that if function is convex then each local minima is global minima.

3. Give an example of convex optimization problem.

4. What is lowener ordering?

5. Give an example of semidefinite programming problem.

6. Weather union and intersection of two convex sets are convex.Justify with examples.

7. True and False:

   (a) $2C \subset C + C$

   (b) $2C = C + C$

   (c) All affine sets are convex set.

   (d) All convex sets are affine set.

   (e) $\mathbb{R}^n$ is not convex cone.

   (f) $H = \{x \in \mathbb{R}^n : \langle a, x \rangle \leq b\}$ is convex cone.

8. State Caratheodary theorem.

9. Prove that a cone C is convex iff for any $x, y \in C,\ x + y \in C$

10. Give an example of polyhedral set.

11. Give an example of proper convex function.

12. Is $\nabla^2 f > 0$ is necessary and sufficient condition for strictly convex function,if not give an example.

13. Prove that $f(x) = \frac{1}{2}\langle x, Qx \rangle + \langle c, x \rangle + d$ is convex where Q is positive semidefinit matrix.

14. If $C_1$ , $C_2$ are closed set then (i) $C_1 + C_2$ is closed? (ii) $C_1 - C_2$ is closed? Justify your claim with examples.

15. Prove or disprove: $f(x) = \frac{1}{2} \parallel y - x \parallel^2$ is a strongly convex function find $\rho > 0$.

16. State John optimality condition.

17. Give an example of pseudoconvex function.

18. Write this minimization problem in the form of variational inequality,

$$\begin{gathered} \min f(x) \\ x \in \mathbb{R}^n \end{gathered}$$

19. What is the necessary and sufficient condtion of optimality in the form of subdifferental of minimizing f(x)?

20. State Rocafellar Pschenichenyi Condition.

21. Sketch the graph in matlab and see the epigraph

$$f(x) = \begin{cases} -\sqrt{1-x^2}, & \text{if } |x| \leq 1 \\ +\infty, & \text{if } |x| > 1 \end{cases}$$

22. If f(x)=max$\{f_1(x), f_2(x), ..., f_n(x)\}$ where each $f_i's$ are convex then f(x) is convex.

23. State the Slater Condition.

24. Write down Karush-Kuhn Tucker optimality condition for a convex programming problem.

25. Prove that for any $v \in N_C(\bar{(x)})$, $\langle v, z\rangle \leq 0 \; \forall \; z \in clcone(C - \bar{x})$

26. Write down KKT condition for the problem

$$\begin{gathered} \min f(x) \\ g_i(x) \leq 0, \;\; i = 1, 2, .., m \\ \text{Ax=B;} \end{gathered}$$

27. State Young-Panchel Inequality.

28. If f is proper and lower semi continuous, then $f(x) = f^{**}(x), \forall x$

29. Fill the blanks:

    (a) If $f(x) = \frac{x^2}{2},\ x \in \mathbb{R}$, then $f^*(x^*) =$ .......

    (b) If $f(x) = \frac{\|x\|^2}{2},\ x \in \mathbb{R}$, then $f^*(x^*) =$ ........

    (c) $f(x) = \delta_C(x),\ C - closed\ set$, then $f^*(x^*) =$ .........

30. Prove that:

    (a) $\sigma_C(x)$ is convex.

    (b) $\sigma_C(\lambda x^*) = \lambda \sigma_C(x^*)$

    (c) $\sigma_C(0) = 0$

    (d) $\sigma_C(x^* + y^*) \leq \sigma_C(x^*) + \sigma_C(y^*)$

31. Give an example where intersection of two convex sets is empty.

32. What will be lagrangian function for the problem

$$\min f(x)$$
$$g_i(x) \leq 0,\ i = 1, 2, .., m$$

33. Fill the blanks:

    (a) $f(x) = 0$ then domf=.....,$f^*(x^*) =$ ......

    (b) $f(x) = e^x$ then domf=.....,$f^*(x^*) =$ ......

    (c) $f(x) = -logx$ then domf=.....,$f^*(x^*) =$ ......

    (d) $f(x) = \sqrt{1 + x^2}$ then domf=.....,$f^*(x^*) =$ ......

34. State Lagrangian Duality Condition for the problem:

$$\min f(x)$$
$$g_i(x) \leq 0,\ i = 1, 2, .., m$$

35. State Weak Duality Condition.

36. Write down the dual of the problem,

$$\min\langle C, X\rangle$$
$$s.t.\ \langle A_i, X\rangle = b_i,\ \ A_i \in S^n$$
$$x \in S^n_+$$

37. Prove that $S^n_+$ is not a polyhedral set.

38. True and False:

    (a) $e^{-x}$ has minimizer at x=0.

    (b) $f(x) = 1/x, x > 0$ does not attains its minimum value.

39. Give an example where Val(CP)≠Val(DP)

40. Write down the transportation problem.

41. Draw the feasible set of this problem,What is the direction of steepest descent for the problem?

$$\min -2x_1 - x_2$$
$$x_1 + x_2 \leq 5$$
$$2x_1 + 3x_2 \leq 12$$
$$x_1 \leq 4$$
$$x_1 \geq 0, x_2 \geq 0$$