Question Bank

Q1. Why for a polyhedral set the extreme points are finite?
Q2. Prove that a bounded polyhedral set is nothing but the convex hull of its vertices.
Q3. Let $A = \{x \in \mathbb{R}^2 : x = (x_1, x_2), (x_1 - 1)^2 + x_2^2 \leq 1\}$. Then calculate Cone(A).
Q4. prove that for an LP optima is obtained at a vertex and thus every minimum is a bfs.
Q5. Check if the following LP will have unique solution or not without calculating it.

$$\begin{gathered} \min\ x_1 + 5x_2 \\ \text{s.t. } x_1 + x_2 \leq 2 \\ x_1 \leq 1 \\ x_1 + 8x_2 \leq 4 \\ x_1, x_2 \geq 0 \end{gathered}$$

Q6.Check if the following LP problem is unbounded or not.

$$\begin{gathered} \min\ x + 2y \\ \text{s.t. } 3x + 4y \geq 9 \\ 2x + 3y \geq 4 \\ x \geq 0, y \geq 0 \end{gathered}$$

Q7. Write down the relaxed KKT condition for the following LP problems.

(i)min $x_1 + 5x_2$
s.t. $x_1 + x_2 \leq 2$
$x_1 \leq 1$
$x_1 + 8x_2 \leq 4$
$x_1, x_2 \geq 0$

(ii)min $x + 2y$
s.t. $3x + 4y \geq 9$
$2x + 3y \geq 4$
$x \geq 0, y \geq 0$
Q8. Prove that $\log(1 + x) \leq x$ if $x \geq -1$.
Q9. What is the LP problem corresponding to the convex optimization problem

min f(x)
$x \in C$
where $f : \mathbb{R}^n \to \mathbb{R}$ is convex and C is a closed and convex set.
Q10. Why S_+^n is not a polyhedral?
Q11. Prove that the dual of an SDP is also an SDP.
Q12. Prove that $\partial\|0\| = B$(Ball of unit radius)
Q13. What is the slater condition for the following optimization problem

$$\begin{aligned} &\text{min f(x)} \\ \text{s.t. } &g_i(x) \le 0 \;\; \forall\, i = 1, 2, ..., m \\ &x \in \mathbb{R}^n \end{aligned}$$

Q14. What is the necessary condition for $\bar{y}$ being a ϵ-solution of the problem

$$\begin{aligned} &\text{min f(x)} \\ \text{s.t. } &g_i(x) \le 0 \;\; \forall\, i = 1, 2, ..., m \\ &x \in \mathbb{R}^n \end{aligned}$$

Q15. What is the set of descent direction of the function $f(x) = (x, e^{-x})$ at the point $x = 0$
Q16. Prove that the set of descent direction of a function f at any point x is an oprn cone.
Q17. Prove that for a convex function the local minimizer is the global minimizer.
Q18. Give an example to show that $\partial h(x) \subseteq \partial g(x)$ is only the necessary condition not sufficient for x being the global minimizer of the D.C. function f=g-h.
Q19.What is the necessary and sufficient condition for $\bar{x}$ to be the ϵ-minimizer of f over a closed, convex set C?
Q20. Prove that $\partial_\epsilon \partial_c(x) = N_c^\epsilon(x)$.