Unit 12 - Week 10: Introductory Stochastic Calculus (Part 2)

Assignment 10

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-10-09, 23:59 IST.

1) Let $[x]$, for $x \in [0, \infty)$, denote the greatest integer less than or equal to x. *1 point*

Then, which of the following functions (defined on $[0, \infty)$) can represent a sample path of a simple process?

- $f_1(x) = [x]$
- $f_2(x) = [x]/2$
- $f_3(x) = [x]/x$
- $f_4(x) = e^{[x]}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$f_1(x) = [x]$
$f_2(x) = [x]/2$
$f_4(x) = e^{[x]}$

2) *2 points*

If the stochastic process $\{X_t\}$ satisfies $dX_t = -aX_t dt + \sigma dW_t$, $X_0 = 0$, where a, σ are positive constants, and $\{W_t\}$ is a Brownian motion, then which of the following is/are true?

- $X_t \sim \mathcal{N}\left(0, \frac{\sigma}{2a}(e^{2at} - 1)\right)$
- $X_t \sim \mathcal{N}\left(0, \frac{\sigma^2}{2a}(1 - e^{-2at})\right)$
- X_t can take negative values.
- $\lim_{t\to\infty} E(X_t) = \frac{1}{a}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$X_t \sim \mathcal{N}\left(0, \frac{\sigma^2}{2a}(1 - e^{-2at})\right)$
X_t can take negative values.

3) State whether the following statement is TRUE or FALSE: *1 point*

If $I_t = \int_0^t s dW_s$, then the quadratic variation of I_t equals the variance of I_t.

- TRUE
- FALSE

No, the answer is incorrect.
Score: 0

Accepted Answers:
TRUE

4) State whether the following statement is TRUE or FALSE: *1 point*

The quadratic variation of an Ito process is always equal to the quadratic variation of the corresponding Ito integral process.

- TRUE
- FALSE

No, the answer is incorrect.
Score: 0

Accepted Answers:
TRUE

5) *2 points*

Let $I_t = \int_0^t X_s dW_s$, where X_t takes the value 1 for $0 \le t < 2$, takes the value 3 for $2 \le t \le 3$ and takes the value 0 otherwise.

Then, which of the following is/are true?

- $I_3 \sim \mathcal{N}(0, 6)$
- I_1 follows a standard normal distribution.
- The distribution of I_2 can not be be determined.
- $\{I_t\}_{t\ge 0}$ is a martingale.

No, the answer is incorrect.
Score: 0

Accepted Answers:
I_1 follows a standard normal distribution.
$\{I_t\}_{t\ge 0}$ is a martingale.

6) State whether the following statement is TRUE or FALSE: *1 point*

The process $\{W_t^3 - tW_t\}$ is a martingale.

- TRUE
- FALSE

No, the answer is incorrect.
Score: 0

Accepted Answers:
FALSE

7) State whether the following statement is TRUE or FALSE: *2 points*

For $X_t = tW_t$ and $Y_t = e^{W_t}$, we have that $d(X_t Y_t) = e^{W_t}\left[\frac{tW_t}{2} + W_t + t\right] dt + te^{W_t} dW_t$.

- TRUE
- FALSE

No, the answer is incorrect.
Score: 0

Accepted Answers:
FALSE