

Unit 11 - Week 9: Introductory Stochastic Calculus (Part 1)

Course outline

How to access the portal?

Week 0

Week 1: Introduction to Financial Markets and Instruments

Week 2: Time Value of Money and Riskfree Assets

Week 3: Modern Portfolio Theory (Part 1)

Week 4: Modern Portfolio Theory (Part 2)

Week 5: Fundamentals of Derivatives

Week 6: Derivative pricing by replication in binomial model

Week 7: Risk-Neutral Pricing in Discrete-Time (Part 1)

Week 8: Risk-Neutral Pricing in Discrete-Time (Part 2)

Week 9: Introductory Stochastic Calculus (Part 1)

Lec 25: General Probability Spaces, Expectations, Change of Measure

Lec 26: Filtrations, Independence, Conditional Expectations

Lec 27: Brownian Motion and its Properties

Quiz : Assignment 9

Feedback Form

Solution: Assignment 9

Week 10: Introductory Stochastic Calculus (Part 2)

Week 11: Risk-Neutral Pricing in Continuous-Time (Part 1)

Week 12: Risk-Neutral Pricing in Continuous-Time (Part 2)

Text Transcripts

Live Session

Assignment 9

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-10-02, 23:59 IST.

1) For the function $f(x) = \sin x, x \in [0, 2\pi]$, which of the following is/are true? 1 point

$f^+(x) = \begin{cases} \sin x, & x \in [0, \pi] \\ 0, & x \in [\pi, 2\pi]. \end{cases}$

$f^-(x) = \begin{cases} 0, & x \in [0, \pi] \\ -\sin x, & x \in [\pi, 2\pi]. \end{cases}$

$f^+(x) = \begin{cases} 0, & x \in [0, \pi] \\ \sin x, & x \in [\pi, 2\pi]. \end{cases}$

$f^-(x) = \begin{cases} 0, & x \in [0, \pi] \\ \sin x, & x \in [\pi, 2\pi]. \end{cases}$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$f^+(x) = \begin{cases} \sin x, & x \in [0, \pi] \\ 0, & x \in [\pi, 2\pi]. \end{cases}$

$f^-(x) = \begin{cases} 0, & x \in [0, \pi] \\ -\sin x, & x \in [\pi, 2\pi]. \end{cases}$

2) 1 point

State whether the following statement is TRUE or FALSE:

If (Ω, \mathcal{F}, P) and (Ω, \mathcal{F}, Q) are two probability spaces such that P and Q are equivalent, then $P(A) > 0$ if and only if $Q(A) > 0$ for all $A \in \mathcal{F}$.

TRUE

FALSE

No, the answer is incorrect.

Score: 0

Accepted Answers:

TRUE

3) Let a sample space be given by $\Omega = C_0[0, 6] = \{\omega \mid \omega : [0, 6] \rightarrow \mathbb{R} \text{ is continuous and } \omega(0) = 0\}$. Then which of the following sets is/are determined by time $t = 4$? 1 point

$\{\omega \mid \min_{0 \leq s \leq 3} \omega(s) \leq -5\}$

$\{\omega \mid \max_{0 \leq s \leq 6} \omega(s) > 4\}$

$\{\omega \mid \omega(4) = 4\}$

$\{\omega \mid \omega(s) = \hat{\omega}(s), \forall s \in [0, 5] \text{ and for some } \hat{\omega} \in \Omega\}$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$\{\omega \mid \min_{0 \leq s \leq 3} \omega(s) \leq -5\}$

$\{\omega \mid \omega(4) = 4\}$

4) Let $T > 0$ and $\{W_t, t \in [0, T]\}$ be a Brownian motion defined on (Ω, \mathcal{F}, P) . 1 point

Then which of the following is/are adapted to the filtration $\{\mathcal{F}_t^W\}$, the filtration generated by the Brownian motion $\{W_t\}$

$\{X_t\}_{t \in [0, T]}$, where $X_t = W_{t+1} - W_t$

$\{Y_t\}_{t \in [0, T]}$, where $Y_t = \max_{0 \leq s \leq t} W_s - W_t$

$\{U_t\}_{t \in [0, T]}$, where $U_t = (W_t - W_2)^+$

$\{V_t\}_{t \in [0, T]}$, where $V_t = W_t^2 - t$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$\{Y_t\}_{t \in [0, T]}$, where $Y_t = \max_{0 \leq s \leq t} W_s - W_t$

$\{V_t\}_{t \in [0, T]}$, where $V_t = W_t^2 - t$

5) Let the two-dimensional random variable (X_1, X_2) have a bivariate normal distribution with parameters $\mu_1 = \mu_2 = 0, \sigma_1^2 = 4$ and $\rho = 0.5$. If the random variable $X_3 = X_2 - X_1$ is independent of X_1 , then the variance of X_3 equals: 1 point

No, the answer is incorrect.

Score: 0

Accepted Answers:

(Type: Numeric) 12

6) Let the two-dimensional random variable (X, Y) have a joint probability mass function given by 2 points

$$f(x, y) = \frac{1}{\pi} e^{-2(x^2 + y^2 - \sqrt{3}xy)}, \quad x, y \in \mathbb{R}.$$

Then which of the following is/are not true?

X and Y are independent.

X and Y are not correlated but independent.

X and Y are correlated and independent

The correlation coefficient between X and Y is 0.75.

No, the answer is incorrect.

Score: 0

Accepted Answers:

X and Y are independent.

X and Y are not correlated but independent.

X and Y are correlated and independent

The correlation coefficient between X and Y is 0.75.

7) Let $\{W_t\}_{t \geq 0}$ be a Brownian motion defined on (Ω, \mathcal{F}, P) .

Then the determinant of the variance-covariance matrix of the three-dimensional random variable (W_1, W_3, W_6) equals:

No, the answer is incorrect.

Score: 0

Accepted Answers:

(Type: Numeric) 6

8) Let $\{W_t\}_{t \geq 0}$ be a Brownian motion defined on (Ω, \mathcal{F}, P) . Then which of the following is/are always true? 2 points

$E(W_5^4) = 75$.

A martingale that has continuous paths and accumulates quadratic variation at the rate of one per unit of time is a Brownian motion.

$\{W_t^2 - t\}$ is a Markov process.

$\{X_t\}_{t \geq 0}$, where $X_t = -W_t, \forall t$, is a Brownian motion.

No, the answer is incorrect.

Score: 0

Accepted Answers:

$E(W_5^4) = 75$.

$\{W_t^2 - t\}$ is a Markov process.

$\{X_t\}_{t \geq 0}$, where $X_t = -W_t, \forall t$, is a Brownian motion.