Unit 11 - Week 9: Introductory Stochastic Calculus (Part 1)

Assignment 9

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-10-02, 23:59 IST.

1) For the function $f(x) = \sin x, x \in [0, 2\pi]$, which of the following is/are true? *1 point*

- $f^+(x) = \begin{cases} \sin x, & x \in [0,\pi] \\ 0, & x \in [\pi, 2\pi]. \end{cases}$
- $f^-(x) = \begin{cases} 0, & x \in [0,\pi] \\ -\sin x, & x \in [\pi, 2\pi]. \end{cases}$
- $f^+(x) = \begin{cases} 0, & x \in [0,\pi] \\ \sin x, & x \in [\pi, 2\pi]. \end{cases}$
- $f^-(x) = \begin{cases} 0, & x \in [0,\pi] \\ \sin x, & x \in [\pi, 2\pi]. \end{cases}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$f^+(x) = \begin{cases} \sin x, & x \in [0,\pi] \\ 0, & x \in [\pi, 2\pi]. \end{cases}$
$f^-(x) = \begin{cases} 0, & x \in [0,\pi] \\ -\sin x, & x \in [\pi, 2\pi]. \end{cases}$

2) *1 point*
State whether the following statement is TRUE or FALSE:
If $(\Omega, \mathcal{F}, P)$ and $(\Omega, \mathcal{F}, Q)$ are two probability spaces such that P and Q are equivalent, then $P(A) > 0$ if and only if $Q(A) > 0$ for all $A \in \mathcal{F}$.

- TRUE
- FALSE

No, the answer is incorrect.
Score: 0

Accepted Answers:
TRUE

3) Let a sample space be given by $\Omega = \mathbb{C}_0[0,6] = \{\omega \mid \omega : [0,6] \to \mathbb{R}$ is continuous and $\omega(0) = 0$. *1 point*
Then which of the following sets is/are determined by time $t = 4$?

- $\{\omega \mid \min_{0 \le s \le 3} \omega(s) \le -5\}$
- $\{\omega \mid \max_{0 \le s \le 6} \omega(s) > 4\}$
- $\{\omega \mid \omega(4) = 4\}$
- $\{\omega \mid \omega(s) = \hat{\omega}(s), \forall s \in [0,5]$ and for some $\hat{\omega} \in \Omega\}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\{\omega \mid \min_{0 \le s \le 3} \omega(s) \le -5\}$
$\{\omega \mid \omega(4) = 4\}$

4) Let $T > 0$ and $\{W_t, t \in [0,T]\}$ be a Brownian motion defined on $(\Omega, \mathcal{F}, P)$. *1 point*
Then which of the following is/are adapted to the filtration $\{\mathcal{F}_t^W\}$, the filtration generated by the Brownian motion $\{W_t\}$

- $\{X_t\}_{t\in[0,T]}$, where $X_t = W_{t+1} - W_t$
- $\{Y_t\}_{t\in[0,T]}$, where $Y_t = \max_{0 \le s \le t} W_s - W_t$
- $\{U_t\}_{t\in[0,T]}$, where $U_t = (W_t - W_2)^+$
- $\{V_t\}_{t\in[0,T]}$, where $V_t = W_t^2 - t$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\{Y_t\}_{t\in[0,T]}$, where $Y_t = \max_{0 \le s \le t} W_s - W_t$
$\{V_t\}_{t\in[0,T]}$, where $V_t = W_t^2 - t$

5) Let the two-dimensional random variable (X_1, X_2) have a bivariate normal distribution with parameters $\mu_1 = \mu_2 = 0, \sigma_1^2 = 4$ and $\rho = 0.5$.
If the random variable $X_3 = X_2 - X_1$ is independent of X_1, then the variance of X_3 equals:

No, the answer is incorrect.
Score: 0

Accepted Answers:
(Type: Numeric) 12

1 point

6) Let the two-dimensional random variable (X, Y) have a joint probability mass function given by *2 points*
$f(x,y) = \frac{1}{\pi} e^{-2(x^2+y^2-\sqrt{3}xy)}, \quad x, y \in \mathbb{R}.$
Then which of the following is/are not true?

- X and Y are independent.
- X and Y are not correlated but independent.
- X and Y are correlated and independent
- The correlation coefficient between X and Y is 0.75.

No, the answer is incorrect.
Score: 0

Accepted Answers:
X and Y are independent.
X and Y are not correlated but independent.
X and Y are correlated and independent
The correlation coefficient between X and Y is 0.75.

7) Let $\{W_t\}_{t\ge0}$ be a Brownian motion defined on $(\Omega, \mathcal{F}, P)$.
Then the determinant of the variance-covariance matrix of the three-dimensional random variable (W_1, W_3, W_6) equals:

No, the answer is incorrect.
Score: 0

Accepted Answers:
(Type: Numeric) 6

1 point

8) Let $\{W_t\}_{t\ge0}$ be a Brownian motion defined on $(\Omega, \mathcal{F}, P)$. Then which of the following is/are always true? *2 points*

- $E(W_5^4) = 75.$
- A martingale that has continuous paths and accumulates quadratic variation at the rate of one per unit of time is a Brownian motion.
- $\{W_t^2 - t\}$ is a Markov process.
- $\{X_t\}_{t\ge0}$, where $X_t = -W_t, \forall t$, is a Brownian motion.

No, the answer is incorrect.
Score: 0

Accepted Answers:
$E(W_5^4) = 75.$
$\{W_t^2 - t\}$ is a Markov process.
$\{X_t\}_{t\ge0}$, where $X_t = -W_t, \forall t$, is a Brownian motion.