Unit 13 - week 11

Assignment 11

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-10-16, 23:59 IST.

Each of the following questions has four options out of which one or more options can be correct. Individual marks are mentioned corresponding to each questions. In case of multiple answers partial marks will be awarded for every correct option chosen provided no incorrect option have been chosen. 0 marks are awarded for questions not attempted.

1) Boys arrive in a queue according to a Poisson process with rate λ_1 and girls arrive in the same queue according to a Poisson process with rate λ_2 independently of the arrival of boys. The probability that first arrival in the queue is a boy is *2 points*

- $\frac{\lambda_1}{\lambda_1+\lambda_2}$
- $\frac{\lambda_2}{\lambda_1+\lambda_2}$
- $1-\frac{1}{\lambda_1+\lambda_2}$
- $\frac{1}{\lambda_1+\lambda_2}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{\lambda_1}{\lambda_1+\lambda_2}$

2) Let $\{X(t), t \geq 0\}$ and $\{Y(t), t \geq 0\}$ be two independent Poisson processes with rates λ_1 and λ_2 respectively. Let $Z(t) = X(t) - Y(t)$. Then *2 points*

- $Z(t)$ need not be a Poisson process.
- $Z(t)$ is a Poisson process with rate $\lambda_1 + \lambda_2$.
- $Z(t)$ is a Poisson process with rate $\min\{\lambda_1, \lambda_2\}$
- $Z(t)$ is a Poisson process with rate $\lambda_1 \lambda_2$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$Z(t)$ need not be a Poisson process.

3) Let mean function $m(t)$ of a renewal process $\{X(t), t \geq 0\}$ be $2t$. Then, $P(X(5) = 2)$ is equal to *2 points*

- e^{-10}
- $50e^{-10}$
- $e^{-\frac{5}{2}}$
- $25e^{-5}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$50e^{-10}$

4) Let $\{N(t), t \geq 0\}$ be a Poisson process with rate λ. Then, its mean function $m(t)$ is *2 points*

- a constant function of t.
- a linear function of t.
- a quadratic function of t.
- a cubic function of t.

No, the answer is incorrect.
Score: 0
Accepted Answers:
a linear function of t.

5) Let $\{X_1, X_2, \ldots\}$ be a sequence of i.i.d. non-negative random variables. Define *2 points*
$S_0 = 0,\ S_n = \sum_{i=1}^{n} X_i,\ n = 1, 2, \ldots$
Let $N(t)$ be defined as $N(t) = max\{n : S_n \leq t\}, t \geq 0$
Then, which of the following is TRUE?

- $N(t) \geq n \iff S_n \leq t$
- $N(t) \geq n \iff S_n \geq t$
- $N(t) = n \iff S_n = t$
- $N(t) \leq n \iff S_n \leq t$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$N(t) \geq n \iff S_n \leq t$

6) Let $\{N(t), t \geq 0\}$ be a renewal process with inter-arrival distribution $U(0, 1)$, i.e., $X_i \sim U(0, 1)$. Let $m(t)$ denote the renewal function of $\{N(t), t \geq 0\}$. Then, which of the following is TRUE for $t \in [0, 1]$? *2 points*

- $m(t) = e^t$
- $m(t) = e^{-t}$
- $m(t) = e^t - 1$
- $m(t) = e^{-t} + 1$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$m(t) = e^t - 1$

7) Consider an age replacement model. Let X be the lifetime of a component with cumulative distribution function F. The component is replaced by a new one upon failure or by time T whichever comes first. The cost of new component is c_1 and cost incurred for replacement is c_2. Let $N(t)$ and $R(t)$ represents the number of components replaced and total cost incurred by time t. Note that $\{N(t), t \geq 0\}$ represent the renewal process and $\{R(t), t \geq 0\}$ is reward renewal process. Let $E(R)$ denotes the expected cost during a cycle, $E(X)$ denotes the expected cycle length. Then, long run average cost is given by *2 points*

- $\frac{E(X)}{E(R)}$
- $\frac{E(R)}{E(R)+E(X)}$
- $\frac{E(X)}{E(R)+E(X)}$
- $\frac{E(R)}{E(X)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{E(R)}{E(X)}$

8) A photographer has a camera that works on a single battery. As soon as the battery in use fails, he immediately replaces it with a new battery. If the lifetime of a battery (in hours) follows uniform distribution $U(5, 55)$, then at what rate does photographer has to change batteries? *2 points*

- $\frac{1}{25}$
- 25
- $\frac{1}{30}$
- 50

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{1}{30}$

9) Which of the following is the property of the time homogeneous Poisson process $\{N(t), t \geq 0\}$ with parameter λ ? *2 points*

- For arbitrary s and t with $0 < s < t$, N(t) - N(s) and N(s) are dependent random variables.
- For fixed t, N(t) follows exponential distribution with parameter λt.
- For arbitrary s and t with $0 < s < t$, $N(t) - N(s)$ follows Poisson distribution with parameter $\lambda(t-s)$.
- N(0) = 1

No, the answer is incorrect.
Score: 0
Accepted Answers:
For arbitrary s and t with $0 < s < t$, $N(t) - N(s)$ follows Poisson distribution with parameter $\lambda(t-s)$.