# Unit 11 - week 9

## Course outline



# Assignment 9

**The due date for submitting this assignment has passed.**
**As per our records you have not submitted this assignment.**

**Due on 2019-10-02, 23:59 IST.**

Each of the following questions has four options out of which one or more options can be correct. Individual marks are mentioned corresponding to each questions. In case of multiple answers partial marks will be awarded for every correct option chosen provided no incorrect option have been chosen. 0 marks are awarded for questions not attempted.

1) Let $\{N(t),\ t \geq 0\}$ be a Poisson process with rate 1. Which of the following stochastic process is martingale with respect to the natural filtration? *2 points*

- $\{N(t), t \geq 0\}$
- $\{N(t)^2 - t, t \geq 0\}$
- $\{(N(t) - t)^2 - t, t \geq 0\}$
- $\{N(t) - 2t,\ t \geq 0\}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\{(N(t) - t)^2 - t, t \geq 0\}$

2) Which of the following statements are TRUE? *2 points*

- Every Markov process is a martingale process.
- The mean of a martingale process always exists.
- The mean of a martingale process need not exists.
- Every stochastic process is a martingale.

No, the answer is incorrect.
Score: 0
Accepted Answers:
*The mean of a martingale process always exists.*

3) Which of the following statements about a stochastic process $\{X(t),\ t \geq 0\}$ is FALSE? *2 points*

- The process $\{X(t),\ t \geq 0\}$ is sub-martingale if and only if the process $\{-X(t),\ t \geq 0\}$ is super-martingale.
- The process $\{X(t),\ t \geq 0\}$ is a martingale implies that the process $\{X(t),\ t \geq 0\}$ is sub-martingale.
- The process $\{X(t),\ t \geq 0\}$ is a martingale if and only if the process $\{X(t),\ t \geq 0\}$ is both a sub-martingale and a super-martingale.
- The process $\{X(t),\ t \geq 0\}$ is a martingale if and only if it is Markov process.

No, the answer is incorrect.
Score: 0
Accepted Answers:
The process $\{X(t),\ t \geq 0\}$ is a martingale if and only if it is Markov process.

4) If $\{W(t), t \geq 0\}$ is a Brownian motion, then which of the following is a martingale? *2 points*

- $\{W(t)^2 - t, t \geq 0\}$
- $\{W(t), t \geq 0\}$
- $\{W(t) - t, t \geq 0\}$
- $\{W(t)^2, t \geq 0\}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\{W(t)^2 - t, t \geq 0\}$
$\{W(t), t \geq 0\}$

5) For what values of $c$, the process $S(t) = c(\sigma + 1)^{N(t)}, \sigma > -1$ is a constant, is a martingale where $\{N(t), t \geq 0\}$ is a Poisson process with rate $\lambda$. *2 points*

- $e^{-\lambda\sigma t}$
- $e^{\lambda\sigma t}$
- $\sigma$
- $\lambda$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$e^{-\lambda\sigma t}$

6) Let $\{W(t), t \geq 0\}$ be a Wiener process. Then the Cov(W(t), W(s)) is given by *2 points*

- t.
- s.
- |t-s|
- min{t,s}

No, the answer is incorrect.
Score: 0
Accepted Answers:
min{t,s}

7) Let $X$ be a random variable and $\mathbb{G}_1$ and $\mathbb{G}_2$ be two sub-$\sigma$-fields of $\mathbb{F}$ such that $\mathbb{G}_1 \subseteq \mathbb{G}_2$. Which of the following statements is true? *2 points*

- $E(E(X \mid \mathbb{G}_1) \mid \mathbb{G}_2) = E(X \mid \mathbb{G}_1)$
- $E(E(X \mid \mathbb{G}_2) \mid \mathbb{G}_1) = E(X \mid \mathbb{G}_1)$
- $E(E(X \mid \mathbb{G}_1) \mid \mathbb{G}_2) = E(X \mid \mathbb{G}_2)$
- $E(E(X \mid \mathbb{G}_1) \mid \mathbb{G}_2) = E(E(X \mid \mathbb{G}_2) \mid \mathbb{G}_1)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$E(E(X \mid \mathbb{G}_1) \mid \mathbb{G}_2) = E(X \mid \mathbb{G}_1)$
$E(E(X \mid \mathbb{G}_2) \mid \mathbb{G}_1) = E(X \mid \mathbb{G}_1)$
$E(E(X \mid \mathbb{G}_1) \mid \mathbb{G}_2) = E(E(X \mid \mathbb{G}_2) \mid \mathbb{G}_1)$

8) Let $X$ be a random variable defined over $\mathbb{F}$ and let $\mathbb{F} = [\emptyset, \Omega]$ be the $\sigma$-field. Then, *2 points*

- $E(X \mid \mathbb{F})$need not exist.
- $E(X \mid \mathbb{F})$always exists.
- $X$ must be a degenerate random variable
- $E(X \mid \mathbb{F})$ always exists and is same as value of $X$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$E(X \mid \mathbb{F})$always exists.
$X$ must be a degenerate random variable
$E(X \mid \mathbb{F})$ always exists and is same as value of $X$.

9) For what values of $c$, the process $S(t) = e^{\mu W(t) - ct\mu^2}$ is a martingale where $\{W(t), t \geq 0\}$ is a Wiener process and $\mu \in R$. *2 points*

- -0.5
- 0.5
- 1
- -1

No, the answer is incorrect.
Score: 0
Accepted Answers:
*0.5*

10) Consider the process $S(t) = \mu t + \sigma W(t)$ where $\{W(t), t \geq 0\}$ is a Wiener process. Which of the following is TRUE? *2 points*

- $S(t)$ is a martingale for every value of $\mu$.
- $S(t)$ is a martingale only if $\mu = 0$.
- $S(t)$ is a martingale for $\mu > 0$.
- $S(t)$ is a martingale for $\mu < 0$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$S(t)$ is a martingale only if $\mu = 0$.