

Unit 6 - week 4

Course outline

How to access the portal?

Assignment 0

week 1

week 2

week 3

week 4

- Introduction to Discrete time Markov Chain
- Introduction to Discrete time Markov Chain (contd.)
- Examples of Discrete time Markov Chain
- Examples of Discrete time Markov Chain (contd.)
- Introduction of Chapman-Kolmogorov equations
- State Transition Diagram and Examples
- Examples
- Introduction to Classification of States and Periodicity
- Closed set of states and irreducible Markov Chain
- First Passage time and Mean Recurrence Time
- Recurrent State and Transient State
- Quiz : Assignment 4
- Assignment 4 Solution
- Feedback Form

week 5

week 6

week 7

week 8

week 9

week 10

week 11

week 12

Assignment 4

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment.

Due on 2019-08-28, 23:59 IST.

Each of the following questions has four options out of which one or more options can be correct. Individual marks are mentioned corresponding to each questions. In case of multiple answers partial marks will be awarded for every correct option chosen provided no incorrect option have been chosen. 0 marks are awarded for questions not attempted.

- 1) Let $\{X_n, n = 1, 2, \dots\}$ be a Markov chain with state space $S = \{1, 2, 3, \dots, 11\}$ such that one-step transition probabilities are given by **2 points**
- $$p_{ij} > 0 \text{ if } |i - j| \text{ is even}$$
- $$p_{ij} = 0 \text{ if } |i - j| \text{ is odd}$$
- Which of the following is TRUE?

- Chain is irreducible
- Chain is aperiodic
- State 5 is transient
- State 3 and 6 belongs to same class

No, the answer is incorrect. Score: 0

Accepted Answers: Chain is aperiodic

- 2) Let $\{X_n, n = 1, 2, \dots\}$ be a Markov chain with state space $S = \{1, 2, 3, \dots, 21\}$ such that one-step transition probabilities are given by **2 points**
- $$p_{ij} > 0 \text{ if } |i - j| \text{ is even}$$
- $$p_{ij} = 0 \text{ if } |i - j| \text{ is odd}$$
- Let $p_{14} > 0$ and $p_{25}^{(3)} > 0$. Which of the following is TRUE?

- Chain is reducible
- Chain is not aperiodic
- State 21 is transient
- State 1 is recurrent

No, the answer is incorrect. Score: 0

Accepted Answers: State 1 is recurrent

- 3) Let X_0 be an integer-valued random variable, $P(X_0 = 0) = 1$, that is independent of the i.i.d. sequence Z_1, Z_2, \dots , where Z_n can take values in the set $\{-1, 1\}$ such that $P(Z_n = -1) = \frac{5}{8}$, $P(Z_n = 1) = \frac{3}{8}$. Let $X_n = X_{n-1} + Z_n$, $n = 1, 2, \dots$. The value of the probability $P(X_5 = 3 | X_2 = 2)$ is equal to **2 points**

- 45/512
- 135/512
- 25/64
- 9/64

No, the answer is incorrect. Score: 0

Accepted Answers: 135/512

- 4) Consider a Markov chain with state space $\{0, 1, 2\}$ and transition matrix $P = \begin{pmatrix} 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \end{pmatrix}$ **2 points**

Which of the following is FALSE?

- The chain is reducible.
- All states are positive recurrent
- The chain is aperiodic
- State 1 is accessible from other states

No, the answer is incorrect. Score: 0

Accepted Answers: State 1 is accessible from other states

- 5) Consider a Markov chain with state space $\{0, 1, 2, 3\}$ and transition matrix $P = \begin{pmatrix} 0.3 & 0.3 & 0.4 & 0 \\ 0.3 & 0.4 & 0.3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ **2 points**

Which of the following is True?

- Markov chain is irreducible
- State 2 is recurrent.
- States 0 and 1 are transient.
- Markov chain consists of only one closed communicating class given by $\{0, 1\}$.

No, the answer is incorrect. Score: 0

Accepted Answers: State 2 is recurrent. States 0 and 1 are transient.

- 6) The transition probability matrix of a discrete time Markov chain $\{X_n, n = 0, 1, \dots\}$ having three states 1, 2 and 3 is $P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ **2 points**

and the initial distribution is $\pi = (0, 1, 0)$. The value of the probability $P(X_{121} = 3, X_{152} = 1)$ is equal to

- 0.22
- 0.5
- 1
- 0

No, the answer is incorrect. Score: 0

Accepted Answers: 1

- 7) Consider a Markov chain with state space $\{0, 1, 2, 3, 4\}$ and transition matrix $P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0.25 & 0.25 & 0 & 0.25 & 0.25 \\ 0 & 0 & 0 & 0.5 & 0.5 \end{pmatrix}$ **2 points**

Which of the following is true?

- All states are recurrent
- States 0, 3, 4 are recurrent and states 2, 4 are transient.
- States 0, 1, 3, 4 are recurrent and state 2 is transient
- Only states 0 is recurrent

No, the answer is incorrect. Score: 0

Accepted Answers: Only states 0 is recurrent

- 8) DTMC with state space $S = \{0, 1, 2, 3, 4\}$ and the one-step transition probability matrix P given by $P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$ Which of the **2 points**

following statements are TRUE?

- State $\{0, 2, 3, 4\}$ are positive recurrent.
- State 0 is null recurrent and $\{2, 3, 4\}$ are positive recurrent.
- State 0 is positive recurrent and $\{2, 3, 4\}$ are null recurrent.
- State 0 is aperiodic and period of state $\{2, 3, 4\}$ is 2.

No, the answer is incorrect. Score: 0

Accepted Answers: State $\{0, 2, 3, 4\}$ are positive recurrent.

- 9) The one-step transition probability matrix of a DTMC $\{X_n, n = 0, 1, 2, \dots\}$ having three states 1, 2 and 3 is $P = \begin{pmatrix} 0.3 & 0.4 & 0.3 \\ 0.6 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.1 \end{pmatrix}$ and the **2 points**

initial distribution is $\pi(0) = (0.7, 0.2, 0.1)$. Then, the value of $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$ is

- 0.0016
- 0.0032
- 0.0072
- 0.0024

No, the answer is incorrect. Score: 0

Accepted Answers: 0.0016

- 10) Let $\{X_n, n = 0, 1, 2, \dots\}$ be a DTMC with state space $\Omega = \{0, 1, 2, 3, 4\}$ and one-step transition probability matrix **2 points**

$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0.75 & 0.25 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$ Which of the following is NOT TRUE?

- Chain is irreducible.
- period of state 1 is 2
- State 1 is recurrent and State 2 is transient.
- period of state 2 is 2.

No, the answer is incorrect. Score: 0

Accepted Answers: Chain is irreducible. period of state 2 is 2.